An estimation and synchronization method based on a new modeling approach of power electrical signals

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1. Introduction

Distributed generation (DG) systems are becoming more common as a result of the increased demand for electricity and the requirement to reduce the impact on the environment from traditional fossil and nuclear sources of power production.

DG systems are relatively small and many of them make use of renewable energy such as photovoltaic, fuel cell, microturbine or small wind power. They have a reverse power flow capability and operate in parallel with utility grid by means of intelligent power interfaces. The correct operation of these interfaces (very often grid-connected inverters) is assured by information about frequency, phase and amplitude of the utility voltage and current. Moreover, an accurate and fast detection of the above quantities is also one of the most important issues regarding the development of power conditioning equipments such as Uninterrupted Power Systems and Series or Shunt Compensators (SC).

On the other hand, especially considering the capacity of SCs and DG systems to operate services as power management or voltage control at distribution level, one can easily understand the importance of developing new approaches able to represent grid voltages and currents both in the steady state conditions as well as in the transient ones.

For this reason, the authors have recently presented a new modeling approach for power electrical signals.

The approach is based on the concept of second-order contact between two curves in the differential geometry, leading to the definition of the osculating circle of a curve at a point.

In a similar way, it is possible to define as “osculating sinusoid” of a generic signal \( y(t) \) at instant \( t_0 \) a special sinusoid, of angular frequency \( \omega_0 \), phase \( \phi_0 \) and amplitude \( V_0 \), satisfying suitable conditions.

Denoting \( \theta(t_0) = \omega_0 t + \phi_0 \), the parameters of this “osculating sinusoid” can be obtained by the following relationships:

\[
\omega_0 = \sqrt{\left\{ \frac{f''(t_0)}{f'(t_0)} \right\}}, \quad (1)
\]

\[
\theta(t_0) = \arctg \left( \frac{\omega_0 y(t_0) / y'(t_0)}{1} \right), \quad (2)
\]

\[
V_0 = \sqrt{\left\{ y(t_0) \right\}^2 + \left\{ y'(t_0) / \omega_0 \right\}^2}. \quad (3)
\]

Because in some instants the quotient included in (1) is ill defined due to an indetermination of the form 0/0, it can be convenient to use a computation algorithm based on the knowledge also of the third order derivative of the signal at instant \( t_0 \).

2. A New Algorithm for Noisy Signals

A problem in applying the above modeling approach is actually due to the fact that the equations used to determine the parameters of the “osculating sinusoid”, require the knowledge of the signal \( y(t) \) and its derivatives up to third order for every instant of time. Indeed, it is known that the measure of an electrical signal contains at least the acquisition noise.

With the aim of overcoming this problem, this paper proposes a new computation algorithm, which takes into account the presence of noise on signal. Consequently, this algorithm, applied to the modeling approach of the power electrical signals, allows to obtain a method providing a fast and accurate estimation of the frequency, phase and amplitude for noisy signals.

Assuming the real noisy signal as \( r(t) = y(t) + n(t) \), where \( n(t) \) is an unstructured perturbation and \( y(t) \) is the noise-free signal, the proposed algorithm is divided in three steps:

**Step 1**: estimation of the angular frequency

\[
\omega_{0e}^2 = 2 \int_{t_0-T}^{t_0} \left[ \frac{T - \tau}{(T - \tau)^2 - 4(T - \tau) \tau + \tau^2} \right] r(\tau) d\tau - 2 \int_{t_0-T}^{t_0} (T - \tau) \tau^2 r(\tau) d\tau, \quad (4)
\]
where $T$ is the size of sliding estimation window.

**Step 2** : estimation of the angle phase

$$
\theta_e(t_0) = \arctg \left( \frac{y_e(t_0)}{y_e^{(1)}(t_0)} \right).
$$

(5)

where $y_e(t_0)$ and $y_e^{(1)}(t_0)$ denote the estimations of the noise-free signal and its first derivative respectively at instant $t_0$ and they are obtained by:

$$
y_e(t_0) = \frac{3}{T^3} \int_{t_0-T}^{t_0} q(\tau)r(t_0-\tau)d\tau.
$$

(6)

$$
y_e^{(1)}(t_0) = \frac{60}{T^5} \int_{t_0-T}^{t_0} p(\tau)r(t_0-\tau)d\tau.
$$

(7)

with

$$
q(\tau) = 3(T-\tau)^2 - 6(T-\tau)\tau + \tau^2,
$$

$$
p(\tau) = 2(T-\tau)^3 - 14(T-\tau)^2\tau + 11(T-\tau)\tau^2 - \tau^3.
$$

**Step 2** : estimation of the amplitude

$$
V_{0e} = \sqrt{ \left[y_e(t_0)\right]^2 + \left[y_e^{(1)}(t_0)\right]^2 / \omega_0 e}.
$$

(8)

### 3. Simulation experiments

In order to explain the performances of the proposed method both during stationary and transient conditions, some significant numerical experiments have been effected. In these experiments the noise level measured by the Signal to Noise Ratio in dB i.e.:

$$
SNR = 10 \log_{10} \left( \frac{\sum y(t)^2}{\sum y(t)^2} \right)
$$

has been taken equal to 40 dB; a size $T = 2 \cdot 10^{-3}$ s , with 2000 samples has been chosen.

The first experiment allows to evaluate the performances of the angular frequency estimation obtained by (4). Actually, the quantity $f_0e = \omega_0 e / 2\pi$ has been taken into account; therefore, an abrupt change of the frequency of the signal, from 50 to 51 Hz, has been considered.

Fig. 2 shows the frequency variation (in blue) and the estimation obtained by the proposed method (in red). As it is evident, after the step variation, the correct value of the frequency is estimated in less than an half of the signal period.

In order to highlight the performance of the method as regards its ability to provide a good synchronization, the second experiment considers a ramp variation from 50 Hz to 60 Hz for the frequency of the signal. Fig. 3 shows as the estimation of the angle phase $\theta(t)$ is accurate and there is always synchronization between the estimated angle phase (in red) and the real signal (in blue), also when the frequency is gradually changing.

The third simulation experiment aims to highlight the ability of the proposed method to provide a correct estimation of all the signal parameters. Therefore, this experiment considers a variation of the signal amplitude which goes from 7 to 5 Volt and lasts about 90 ms. Fig. 4 shows again the comparison between real signal (in blue) and the estimated angle phase (in red) and also that between the real signal amplitude variation (in green) and its estimation (in black). The figure highlights the good estimation of the signal amplitude obtained also during the transient condition considered, without losing synchronization between the estimation of the angle phase $\theta(t)$ and the real signal.

![Fig. 2. Frequency estimation](image1)

![Fig. 3. Angle phase estimation](image2)

![Fig. 4. Angle phase and amplitude estimations](image3)