Online Determination of Grid Impedance Spectrum through Pseudo-Random Excitation of a Pulse Width Modulator

Surena Neshvad, Symeon Chatzinotas, Jürgen Sachau
Interdisciplinary Centre for Security, Reliability and Trust (SnT)
University of Luxembourg
Luxembourg City, Luxembourg
e-mail: Surena.Neshvad@uni.lu, Symeon.Chatzinotas@uni.lu, Juergen.Sachau@uni.lu

Abstract. The last decade has seen a dramatic rise in renewable energy converters connected to the grid, consisting mostly of intermittent distributed generators on the medium or low voltage grid. Due to this evolution in energy transmission, improved monitoring and control of the distribution grid is becoming mandatory in order to efficiently integrate the new power sources and guarantee power quality, efficiency and reliability. Our paper describes a reliable and precise power grid impedance estimation method using an innovative scheme to control the Pulse Width Modulator’s pulse pattern on the inverters, in order to inject broad spectrum identification patterns. The generated harmonics and inter-harmonics will allow the online computation of the complete spectrum of the grid impedance at the Point of Common Coupling during normal operation. Simulations in typical grid situations verify that the proposed algorithm is robust to a realistic environment and can be used for automated feedback control.

Key words
Grid Impedance; System Identification; Pulse Width Modulation; Grid Tie Inverter; Pseudo Random Binary Sequence

1. Introduction

This last decade brought a substantial growth in renewable, distributed energy production in industrial countries. Due to the ‘20% renewables by 2020’ goal targeted by the European Community, this trend will accelerate and Distributed Generation (DG) will represent a significant portion of the European energy mix. A large part of the new renewable energy sources consists of wind and photovoltaic energy (PV). As these are by nature variable energy sources, careful planning is required to integrate them harmoniously and efficiently. The improved sustainability of energy supply requires optimized regional grid use and increased reliability and security of supply. While the High Voltage (HV) Grid control mechanisms are well established, the Medium Voltage (MV) Grid, where a large amount of DG is feeding in its energy, is sparsely observed and subjected to power quality and stability issues. In order to address these concerns, the MV network must be tightly monitored. However replicating the High Voltage infrastructure on the MV would require large investments. With the shift to renewable energy and wide spread inverter based generators, it is crucial to integrate a real-time monitoring of the relevant parameters of the grid at key points. An online monitoring of the grid impedance allows the operators to react instantaneously to critical conditions and enables the correct behavior of DG.

An impedance analysis on the complete harmonic spectrum reveals properties of the grid that are hard to detect through traditional methods. Islanding conditions are situations during which a distributed generator will feed a local sub-grid, while the main grid is shut down [1]. Such situations are dangerous if not detected promptly, and can be tracked continuously through sudden impedance variations. The grid impedance spectrum also indicates the resonance frequencies in the network and major harmonics induced by the load capacitance [2]. In [3], the impedance is used for the inverters droop control optimization, by adjusting the amplitude and the frequency of the inverter output voltage for controlling active and reactive power delivery. The research in [4] discusses the importance of the grid impedance to assess the voltage stability of the grid tie inverter and suggests that it is crucial to enforce defined conditions on the ratio of the voltage output impedance and the grid to guarantee stability.

Various techniques can be used for impedance measurement, and these typically require dedicated hardware, consisting of a signal injector, voltage and current measurement device followed by signal processing performed on a digital controller. Online measuring methods can be categorized in two main branches: passive and active system identification. Passive methods rely on non-fundamental noise present in the grid, e.g. transient noise and converter noise. [5] uses the switching frequency of the inverters active-shunt-filter (ASF) to act as a naturally occurring harmonic, and estimates the impedance spectrum.
The Pseudo-Random Binary sequences are bit streams of ‘1’s and ‘0’s occurring randomly, but in a predefined manner. They have unique properties that make them very suitable for excitation signals in characteristics analysis and system identification. They are generated by a series of shift registers, combined with logical XOR gates. The number of shift registers and the position of the XORs determine the run length of the series and are defined by its polynomial, as shown in Figure 1. Thus, for a given seed, a polynomial of length \( L \) will allow the generation of a deterministic pseudo-random binary sequence of ‘1’s and ‘0’s of \( 2^L \) elements.

Several properties of the PRBS sequence make it an ideal candidate for grid identification:

**Spectrum:** Depending on the relationship between the code-length, sampling frequency and code frequency, the PRBS exhibits a white noise like spectrum for a certain frequency range, with zeros occurring at multiples of the PRBS clock sampling frequency. For impedance determination, a high amplitude spectrum is necessary in the desired range to achieve optimal energy distribution and accuracy. This has to be mitigated with the injected THD in order to find the adequate compromise between Signal-to-Noise Ratio (SNR) and harmonic pollution. The code length and its sampling frequency limit the resolution of the impedance spectrum that will be excited. The objective is to enable an injection method that will cover the desired range, with the hardware constraints of an inverter.

This paper investigates an impedance measurement method through the use of Pseudo-Random-Binary-Sequences (PRBS). These patterns have thoroughly been used in various engineering fields e.g. telecommunications and information theory, and their inherent properties suit well for system identification. The purpose of our impedance identification technique is to improve upon existing methods on several aspects. Firstly, our method requires minimal hardware addition to currently deployed infrastructure. For some, the Total Harmonic Distortion (THD) and crest factor injected are too high for grid codes such as EN 50160 [15], the rules enforced by CREOS on the Luxemburgish distribution network. Thus, scaling the aforementioned implementations on a large number of inverters in fragile networks e.g. weak high-impedance grids could render stability problems.

This paper is structured as follows: Section 2 presents a description of the PRBS patterns, depicts their implementation method, and details their spectral and temporal properties. Section 3 portrays a typical grid-tie inverter setup, whose Pulse-Width Modulation (PWM) control has been modified in order to overlay PRBS patterns on top of the fundamental. The complete setup for the impedance measurement and the calculation are presented in Section 4. In order to validate the theoretical calculations, Section 5 illustrates simulations and analysis.

### 2. Pseudo-Random Binary Sequence Properties

![Fig. 1. A Pseudo Random Generator with polynomial 1+x^3+x^4, generating a repeated random sequence of 15 bits](image)

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**White noise behavior:** The PRBS exhibits white noise-like properties, and thus correlation techniques, eliminating measurement noise described in [9], can be utilized for identification purposes.

**Ease of generation and implementation:** The digital logic required for PRBS implementation consists of shift registers and XOR gates and can be implemented on a very modest digital controller.

**Low crest factor:** Compared to other active injection techniques mentioned in this paper, the PRBS is the one most likely to pass grid codes such as EN 50160. In fact, its operational crest factor is very low and the instantaneous THD is significantly less than other active identification methods. This is accomplished without compromising accuracy, since the low amplitude pulses, which are aggregated over the complete sequence, provide sufficient energy for identification.

**Easily Customizable:** The PRBS can easily be lengthened or have its amplitude increased so as to improve SNR and spectrum range. Thus, a short sequence could be used for coarse estimation, and a longer one for a more refined spectrum.

### 3. PRBS on a Single Phase Inverter

#### A. Grid connected inverter

The system topology is depicted in Figure 2. It comprises a DC Voltage source, with a pulse-width modulation Voltage Source Inverter (VSI) connected to the grid through a low pass LCL-filter. The impedance of the grid is modeled by an impedance $Z_g$, consisting of a resistive component $R_g$ and an inductive component $L_g$. While more complex grid models exist, for our purposes, a basic series inductive-resistive impedance, combined with an ideal source supply, is sufficient. Typical modern grid tie inverters operate with LCL filters, due to their superior filtering capabilities at higher frequencies [10]. The main issue of the LCL filter is its resonance frequency induced by the capacitance in parallel between the inductors. Here, in order to reduce the gain of the resonance, a damping resistor is connected in series with the capacitor. This setup limits the currents passing through the capacitor and attenuates the voltage gain at PCC for the resonant frequency.

An analytical model of the system has been established in [11]. The resonance frequency of the LCL filter is expressed by:

$$f_{res} = \frac{1}{2\pi} \sqrt{\frac{L_{f1} + L_{f2} + L_g}{L_{f1} \cdot (L_{f2} + L_g) \cdot C_f}} \quad (1)$$

From (1) it can be seen that the resonant frequency of the LCL filter is dependent on the grid impedance and an increase in the grid inductance triggers a decrease of the resonant frequency. Thus, in typical implementations, the system is designed to have the resonant in the range

$$f_g \ll f_{res} < f_{PWM}$$

This constraint prevents its interference with the fundamental and the PWM switching frequency. As described in the Section 4, it will amplify the excitation signals over the range $[0.5 \cdot f_{res}, 2 \cdot f_{res}]$, and improve identification in that range.

#### B. PRBS implementation on PWM

A common PWM architecture for a single phase grid-tie inverter is depicted in Figure 2. It uses high, low and zero voltage levels. The resulting PWM signal is used to steer one half of an H-bridge. The other half of the H-bridge controls the polarity of the voltage across the load, and is triggered by a simple square wave of the same frequency and in phase with the sine signal. Further details about the structure and functionality of a three level PWM can be obtained in [12].

In order to implement the PRBS pattern, the carrier shape is altered so that the PWM naturally overlaps the PRBS with the reference signal to be generated. As shown in Figure 3, a ‘1’ PRBS code corresponds to a slightly elevated triangle peak for one pulse, and a ‘0’ PRBS code corresponds to a lowered peak. The original PWM response is depicted in blue in Figure 3, the altered one in red. The effect on the pulse train is that ‘1’ pulses are slightly widened compared to the original PWM, ‘0’ pulses slightly narrowed. Using these constraints, the PRBS code frequency will be limited to the inverters switching frequency. Its spectrum, shown in Figure 4, is typically wider than the frequency range needed for identification purposes.
switching frequency being redirected to the intermediate frequencies. Figure 4-B shows the spectrum of $V_{\text{PCC}}$, behind the LCL-filter. The high frequency harmonics are dampened extensively, and beyond $3\times f_{\text{ref}}$, the generated excitation is not sufficient for adequate impedance detection. In the simulation settings $f_{\text{ref}}$ will be set to 1263 Hz according to equation (1) and the grid and filter parameters set in Table II.

The amplitude of the PRBS spectrum depends on the magnitude of the alteration of the carrier peaks. Simulation results show that even minimal alterations provide excellent results, since slight pulse modifications of the PRBS are aggregated over many carrier pulses. Thus longer PRBS periods can be used to increase SNR.

4. Grid Impedance Determination

The setup of this research is described in Figure 2. In normal operation, the DG inverter, operating at 12.8 kHz, provides active and reactive power to the grid, and its spectrum, shown in Figure 4, contains the fundamental 50 Hz and harmonics consisting mainly of the inverter switching. Periodically, during four fundamental cycles, a 1024 bit PRBS burst is injected, modulated on top of the 50 Hz sinusoid, according to the method detailed in the previous section. During the burst, the spectrum of the PRBS is overlapped over the naturally generated harmonics. Using the $V_{\text{PCC}}$ and $I_{\text{PCC}}$ measured at the point of common coupling of the inverters power control loop, the complex impedance is calculated by the voltage to current ratio:

$$Z_g(h) = \frac{V_{\text{PCC}}(h)}{I_{\text{PCC}}(h)}$$ (2)

$$Z_g(h) = \frac{V_{\text{PCC}}(h) \cdot e^{j\varphi(h)}}{I_{\text{PCC}}(h) \cdot e^{j\varphi(h)}}$$ (3)

In the equations above, $V_{\text{PCC}}(h)$ and $I_{\text{PCC}}(h)$ represent the complex line voltage and current at a given frequency. Their magnitude is $V_{\text{PCC}}(h)$ and $I_{\text{PCC}}(h)$ and phase $\varphi(h)$ and $\varphi(h)$. $Z_g(h)$ is the complex grid impedance at a given frequency, its amplitude is $Z_g(h)$ and its phase $\varphi(h)$. The objective is to obtain the grid impedance at PCC for all harmonics and inter-harmonics in order to get the complete spectrum. Thus:

$$Z(f) = \frac{DFT\left(v_{(t)}\right)}{DFT\left(i_{(t)}\right)} = R_{\text{grid}} + j \times X_{\text{grid}}$$ (4)

where ‘DFT’ denotes the Fourier transform of the time domain measurement of the voltage and current at the PCC. The PRBS codes are aligned and synchronized with the fundamental, and performing the Fourier transform over four full fundamental cycles will render results containing minimal spectral leakage. Furthermore, the presented technique has a latency of 80ms, and is fast.
enough for power grid related applications, which have time constants that are usually larger by at least one order of magnitude. In the next section, analytical data on SNR, THD and impedance accuracy will be discussed using Matlab simulations.

5. Simulations

Simulations were carried out using Matlab with Simulink. A detailed discrete-time model of a single phase grid-connected inverter has been considered for the performance of the proposed estimation method. The system structure is depicted Figure 2 and its operating parameters are listed in Table I. The LCL filter parameters are selected according to [11]. A 1024 PRBS code is injected, with the code frequency set to the carrier frequency: 12.8 KHz. The 1024 codes are injected in 80ms and Figure 5 shows a large zoom on the grid voltage’s evolution at PCC for the nominal case and with PRBS injection. The PRBS induced variation tends to be relatively small, the main harmonics are the switching harmonics and a jitter type distortion can be seen overlapped on it with the PRBS mode.

<table>
<thead>
<tr>
<th>Parameter and designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter inductance $L_{f1}$</td>
<td>17.7 mH</td>
</tr>
<tr>
<td>Filter inductance $L_{f2}$</td>
<td>05.7 mH</td>
</tr>
<tr>
<td>Filter capacitance $C_f$</td>
<td>3.45 µF</td>
</tr>
<tr>
<td>Filter damping resistance $R_d$</td>
<td>11.2 Ω</td>
</tr>
<tr>
<td>PWM switching frequency $f_{PWM}$</td>
<td>12.8 KHz</td>
</tr>
<tr>
<td>Vg grid voltage</td>
<td>20.0 kV</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>10e5 Hz</td>
</tr>
<tr>
<td>Grid resistance</td>
<td>0.08 Ω</td>
</tr>
<tr>
<td>Grid inductance</td>
<td>0.4 mH</td>
</tr>
<tr>
<td>PRBS code length</td>
<td>1024</td>
</tr>
<tr>
<td>PRBS carrier amplitude modification</td>
<td>5%</td>
</tr>
</tbody>
</table>

Figure 5: Time domain plot of $V_{PCC}$ – Blue: original Voltage curve, Red: modified Voltage curve with ‘1’,’1’,’0’,’1’ code

A controller with a sampling frequency of 10 KHz has been considered. It encompasses the control model PWM signal generation, inverter behavior and current and voltage sampling with 16-bit quantization. Grid resistance and inductance are set respectively to $R_g = 0.4 \, \Omega$ and $L_g = 0.8 \, mH$, which renders an impedance at the fundamental frequency of $Z_g = 0.89 \, \Omega$.

The calculated impedances from online transfer function identification are depicted in Figure 6 and 7. Figure 6 shows the impedance at PCC. The expected RC spectrum is shown in blue, and the red plot is the result of the real-time identification technique. No smoothing or averaging process has been applied, in order to highlight the performance of the proposed method. The results are quite accurate, and due to the correlation properties of the PRBS, very resilient to noise. Thus, good results can be obtained in difficult conditions, where the amplitude of the injection is much lower than the environmental noise.

Table II shows the harmonic “pollution” injected by the proposed measurement technique for different injection strengths values. Excellent results can be obtained with very little distortion, 10% above the natural distortion present in the PWM harmonics. With higher injection amplitude, the rate of return declines. Finally, investigation has shown that the remaining impedance estimation error is mainly due to quantization error and spectrum leakage and could be
improved by lengthening the PRBS code sequence or applying windowing filters such as the Blackman window. This in turn would deteriorate the THD and execution latency. Therefore, a compromise has to be made in order to find the right balance. A 6% estimation error is very promising and further research and optimization would improve the results even more.

Table II. - THD and Impedance Error for various injection magnitudes

<table>
<thead>
<tr>
<th>No</th>
<th>THD</th>
<th>Impedance Estimation Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>prbs 0.5%</td>
<td>1.67e-4</td>
<td>10.44%</td>
</tr>
<tr>
<td>prbs 1%</td>
<td>1.86e-4</td>
<td>7.29%</td>
</tr>
<tr>
<td>prbs 3%</td>
<td>2.42e-04</td>
<td>7.08%</td>
</tr>
</tbody>
</table>

6. Conclusions

The number of distributed power electronic based generators connected to the grid is increasing and their influence on grid infrastructure, stability and reliability is growing. The real-time knowledge of the equivalent grid impedance at the inverter’s PCC is crucial for filter design, power quality evaluations and grid status determination. An advanced multi-purpose real-time estimation method for frequency dependent grid impedance determination method has been presented in this paper. The method is based on PRBS sequences, which have been extensively used in system identification, communications and information theory. An innovative technique injecting PRBS on the inverter’s Pulse-Width-Modulators has been introduced and a detailed description has been provided. Practical applications such as islanding detection and inverter tuning in distorted grid conditions have been discussed.

The proposed method estimates the equivalent grid impedance over a significant frequency range with a high resolution. It provides a high degree of flexibility; longer injections provide higher frequency resolution and accuracy, shorter patterns reduce latency and THD. The proposed technique will be implemented in a prototype converter at SnT’s Netpower Laboratories in order to verify the performance in real settings and confirms the simulations research. Further research aiming to identify the power network parameters will be carried out as well.

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