Three-phase four wire shunt active power filter based on Simplified Backstepping technique for DC voltage control

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Abstract.
This work presents a simulation of Three-Phase Four-Wire Shunt Active Power Filter (SAPF) destined to suppress harmonic currents generated by nonlinear loads. The SAPF guarantees also the compensation for unbalanced nonlinear load currents, reactive power and harmonic neutral current. The identification method is based on the synchronous reference frame.

In order to have good performances of DC voltage control, simplified Backstepping control for three-Phase four-Wire (SAPF) system is proposed.

Complete simulation of the resultant active filtering system validates the efficiency of the proposed simplified Backstepping control law over to the traditional control with PI controller.

Keywords: DC bus voltage control, Harmonic currents, Simplified Backstepping control, Three-Phase Four-Wire Shunt Active Power Filter (SAPF).

1. Introduction

The use of nonlinear loads connected to the utility knows a fast growth; these nonlinear loads have contributed to the deterioration of the power quality (PQ) by the generation of harmonics and reactive currents which are distorting the voltage at the point of common coupling [1]. Several solutions have been studied to improve the PQ by reducing the harmonic currents in the electrical power system, such as passive filters and active filters [2]. Passive filters cannot totally eliminate all of the harmonic currents and it can present resonance problems [3]. However, in order to cancel the harmonic currents, to compensate reactive power and to improve the filtering performances; the three- and single-phase shunt active power filters (SAPF) and hybrid filter topologies have been used [4], but when nonlinear single-phase load are connected to three-phase, four-wire systems, the problem of circulation of harmonic currents through the neutral conductor occur, in this situation SAPF can also compensate the current unbalances [5].

An important point in the SAPF is the control of DC voltage across the capacitor at a fixed value. This is necessary because there are the active filter losses.

In order to control DC link voltage, several works treated with DC bus controllers, such as P and PI controllers [6]. These controllers require a precise mathematical model of the system, which are difficult to obtain under load disturbance and parameters variation.

In this work, our objective is to design a model based on simplified backstepping controller for DC voltage control allowing a very good dynamic with small response time.

The rest of the paper is organized as follows: the next section, principle of SAPF with mathematical system modeling has been presented. Section 3 presents a controller design of the three phase four wire SAPF using both traditional PI controller and a novel simplified backstepping technique for dc link voltage.
This work is illustrated by simulation works in Matlab/Simulink, and results are discussed in Section 4. Lastly, a precise conclusion has been added in section 5 to finalize the work.

### 2. Three-phase four-wire SAPF: Basic scheme and modeling

Figure 1 shows the scheme that represents the a SAPF topology applied to a three-phase system using a four-leg full-bridge VSI converter \[7\]; the four-leg SAPF is connected in parallel with the AC three-phase four-wire system through four inductors. The inductances \(L_f\) are used to reduce and smooth the ripples of the harmonic currents injected by SAPF and the capacitor is used to stock energy.

![Figure 1: Active power filter: power circuit](image)

In the power circuit shown in Figure 1, \(V_{sx}\) and \(i_{fx}\), \(x = a, b, c\), represent the point of common coupling (PCC) voltages and AC side currents of the SAPF, respectively. \(R_L\) is a line resistance that models the parasitic resistive effects of the inductor \(L_f\).

\[
V_{\text{anpwm}} = R_L i_{fa} + L_f \frac{di_{fa}}{dt} + V_{sa} + L_f \frac{di_{fn}}{dt} + R_L i_{fn} \tag{1}
\]

\[
V_{\text{bnpwm}} = R_L i_{fb} + L_f \frac{di_{fb}}{dt} + V_{sb} + L_f \frac{di_{fn}}{dt} + R_L i_{fn} \tag{2}
\]

\[
V_{\text{cnpwm}} = R_L i_{fc} + L_f \frac{di_{fc}}{dt} + V_{sc} + L_f \frac{di_{fn}}{dt} + R_L i_{fn} \tag{3}
\]

Where \(V_{\text{anpwm}} = V_x - V_n\), \(x = a, b, c\)

As \(i_{fa} + i_{fb} + i_{fc} = i_{fn}\) its derivative form is given by:

\[
\frac{di_{fa}}{dt} + \frac{di_{fb}}{dt} + \frac{di_{fc}}{dt} = \frac{di_{fn}}{dt} \tag{4}
\]

For obtaining the state space system and the transfer functions in the two-phase stationary reference frame (ab0-axes) and in the dq0 frame, the mathematical modeling is presented. To perform the modeling, all the coupling inductances and their resistances are assumed to be identical, such as:

\[L_{a0} = L_{b0} = L_{c0} = L_f\] and \(R_{a0} = R_{b0} = R_{c0} = R_L\f\]

So, the state-space model in the two-phase stationary reference frame (ab0-axes) and into dq0 -axes represented by (5) and (6), respectively can be obtained, \((V_{sa} , V_{sb} , V_{sc} \) and \(V_{sn})\) are not taken into account in the model because it’s considered as disturbances \[8]\[9\].

\[
\begin{align*}
\frac{di_{fa}}{dt} & = \frac{R_L}{L_f} \left[ -1 0 0 \right] \left[ i_{fa} \atop i_{fb} \atop i_{fn} \right] + \frac{1}{4L_f} \left[ 4 0 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] \\
\frac{di_{fb}}{dt} & = \frac{R_L}{L_f} \left[ -1 -1 0 \right] \left[ i_{fa} \atop i_{fb} \atop i_{fn} \right] + \frac{1}{4L_f} \left[ 4 0 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] \\
\frac{di_{fc}}{dt} & = \frac{R_L}{L_f} \left[ 0 -1 -1 \right] \left[ i_{fa} \atop i_{fb} \atop i_{fn} \right] + \frac{1}{4L_f} \left[ 4 0 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] \\
\frac{di_{fn}}{dt} & = \frac{R_L}{L_f} \left[ 0 0 -1 \right] \left[ i_{fa} \atop i_{fb} \atop i_{fn} \right] + \frac{1}{4L_f} \left[ 4 0 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] 
\end{align*}
\tag{5}
\]

\[
\begin{align*}
\frac{d\psi_{anpwm}}{dt} & = R_L \frac{1}{L_f} \left[ -1 0 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] + \frac{1}{4L_f} \left[ 4 0 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] \\
\frac{d\psi_{bnpwm}}{dt} & = R_L \frac{1}{L_f} \left[ -1 -1 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] + \frac{1}{4L_f} \left[ 4 0 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] \\
\frac{d\psi_{cnpwm}}{dt} & = R_L \frac{1}{L_f} \left[ 0 -1 -1 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] + \frac{1}{4L_f} \left[ 4 0 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] \\
\frac{d\psi_{sn}}{dt} & = R_L \frac{1}{L_f} \left[ 0 0 -1 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] + \frac{1}{4L_f} \left[ 4 0 0 \right] \left[ \psi_{anpwm} \atop \psi_{bnpwm} \atop \psi_{cnpwm} \right] 
\end{align*}
\tag{6}
\]

### 3. Control system

In this section, the control system is described in two subsections, as follows:

i. Algorithm for extraction of compensation currents references and ii. Control design.

#### 3. a. Reference currents extraction

In order to extract the three-phase compensation reference currents \(i^*_f a\), \(i^*_f b\) and \(i^*_f c\) and specifically, the neutral reference current \(i^*_n\), the SRF-based algorithms are used to perform load unbalance compensation or to independently compensate each of the three phases.

In order to detect all the harmonic currents \(i_L\) (and \(I_{Lc}\)), the fundamental currents \(i_L\) are subtracted from total load currents \(i_L\) using a SRF-based algorithm. In provision of unbalanced loads connected to a four-wire three-phase system, the fundamental currents are represented by positive-negative and zero-sequence components, such as \(i_L = \begin{bmatrix} \bar{i}_L &= \begin{bmatrix} i_{L1} \cdot i_{L2} \cdot i_{Lc} \end{bmatrix} 

In order to extract each one of the fundamental components \(i^*_1\), \(i^*_1\) and \(i^*_1\) \(i^*_1\) from unique three-phase load which are represented into the synchronous reference frame (dq -axis), the SRF algorithm allows this extraction perfectly with some modification that must be done to apply it to a three-phase system, as can be seen in Figure 2.
In order to detect all of harmonic currents, as it can be seen in the algorithm shown in Figure 2, only the harmonic currents in the load can contribute to the output quantities ($i^{*}_{fa}$, $i^{*}_{fb}$ and $i^{*}_{f0}$) by subtracting ($i_{dc}$ and $i_{d_{dc}}$), ($i^{*}_{fa}$ and $i^{*}_{fb}$) and ($i^{0}_{d_{dc}}$ and $i^{0}_{q_{dc}}$), which are related to fundamental currents [8].

3. b. Control design

3. b. 1. The compensation current control loops

The current control loops are implemented in two-phase stationary reference frame and it can be represented by three independent control loops, Figure 3. According to the model presented in (1)-(4), the three-phase converter acts as three single-phase systems in the αβ0 coordinates (5) [8],[9].

So we can obtain three independent transfer functions as given by (7) and (8),

$$G_{i\alpha,\beta}(s) = \frac{i_{f(\alpha,\beta)}}{V_{f(\alpha,\beta)}}(s) = \frac{1}{R_{L_f} + sL_f} \quad (7)$$

$$G_{i0}(s) = \frac{i_{f(\alpha,\beta)}}{V_0}(s) = \frac{1}{4(R_{L_f} + sL_f)} \quad (8)$$

Similarly, the closed loop transfer function can be obtained for the three current control loops, as given by (9).

$$N_{i_{f(\alpha,\beta)}}(s) = \frac{i_{f(\alpha,\beta)}}{i_{f(\alpha,\beta)}} = \frac{A_2s^2 + A_1s + A_0}{B_1s^2 + B_2s + B_3} \quad (9)$$

Where: $A_0 = V_{dc}K_{pvw}K_{in}$

$A_1 = V_{dc}K_{pvw}[K_{ipr} - K_{in}(\frac{T_s}{4})]$.

$A_2 = -V_{dc}K_{pvw}K_{ipr}B_0 = V_{dc}K_{pvw}K_{in}$.

$B_1 = R_{L_f} + V_{dc}K_{pvw}[K_{ipr} - K_{in}(\frac{T_s}{4})]$.

$B_2 = (\frac{T_s}{4})R_{L_f} - V_{dc}K_{pvw}K_{ipr}(\frac{T_s}{4})$.

$B_3 = (\frac{T_s}{4})L_f$.

3. b. 2. DC link voltage regulation

The DC link capacitor voltage has to be regulated and maintained at a constant value, for satisfying this
objective, the active power flowing into the active filter needs to be controlled. If the active power flowing into the filter can be controlled equal to the losses inside the filter, the DC link voltage can be maintained at the desired value.

A traditional PI and simplified Backstepping controllers are presented in the next section for controlling the DC link voltage.

3. b. 2.1. The DC-bus voltage control by traditional PI controller

The output of the PI controller is considered as peak value of the supply current \( I_{max} \), which is composed of two components: (a) fundamental active power component of load current, and (b) loss component of APF; to maintain the average capacitor voltage to a constant value.

Figure 4 shows the mathematical model for the closed loop transfer function related to the dc-bus voltage control which can be obtained by (10), with \( V_{dc_{ref}} \) is the DC capacitor reference voltage.

\[
N_i(s) = \frac{V_{dc}}{V_{dc_{ref}}} = \frac{V_d + K_{pr} \cdot s + V_d \cdot K_{ix}}{C_{dc} \cdot V_{dc} \cdot s + V_{d_{in}} \cdot K_{pr} \cdot s + V_{dc_{ref}} \cdot K_{in}} \quad (10)
\]

All the proportional and integrative gains (\( K_{pr} \) and \( K_{in} \)) are projected using the methodology of Ziegler-Nichols [10].

![Figure 4: PI DC Bus voltage control](image)

3. b. 2.2. The DC-bus voltage control by simplified Backstepping controller

Recently, Backstepping controllers have been introduced in SAPF control [11]. The advantage of Backstepping controllers over the proportional P controllers is that, it does not need an accurate mathematical model of the system; another advantage of this simplified method is the simplicity of calculations, since only algebraic operations are required.

Based on Lyapunov theory; for a nonlinear system to be stable at the point m, it is necessary and sufficient that there is a function continually derivable \( G(m) \) that satisfies the following three conditions:

\[
G(m) < 0 \quad \forall \ m \neq 0, \ m \in \emptyset \quad \emptyset: \text{domain of study}
\]

To start with, for controlling \( V_{dc} \), we consider the error defined as follows:

\[
e_{r_1} = V_{dc_{ref}} - V_{dc}
\]

We define the energy of the capacitor as follows.

\[
E_{dc}(V_{dc}) = \frac{1}{2} \cdot C \cdot V_{dc}^2 \quad (11)
\]

We define a new term \( T_{dc}(V_{dc}) \) for small variations of \( V_{dc} \)

\[
T_{dc}(V_{dc}) = \dot{E}_{dc}(V_{dc}) = C \cdot V_{dc} \cdot \dot{V}_{dc} \quad \dot{V}_{dc} = \frac{\tau_{dc}(V_{dc})}{C \cdot V_{dc}} \quad (12)
\]

We define the second error as follows.

\[
e_{r_2} = \Delta E_{dc} = \frac{1}{2} \cdot C \cdot e_{r_1}^2 \quad (13)
\]

According to the previous expression, the first and the second conditions are satisfied where \( G = e_{r_2}(V_{dc}) \) and \( m = V_{dc_{ref}} \)

Let us now check the third condition, for this purpose we derive

\[
e_{r_2} = \Delta E_{dc} \quad e_{r_2} = \Delta T_{dc}
\]

\[
e_{r_2} < 0 \iff \Delta T_{dc} < 0 \rightarrow -C \cdot e_{r_1} \dot{V}_{dc} < 0 \quad (14)
\]

From (12) we can write:

\[
-C \cdot e_{r_1} \cdot \frac{\tau_{dc}(V_{dc})}{C \cdot V_{dc}} < 0 \iff -e_{r_1} \cdot \frac{T_{dc}(V_{dc})}{V_{dc}} < 0 \quad (15)
\]

In order to satisfy the third condition, a constant L (Strictly positive) is introduced so that:

\[
\frac{T_{dc}(V_{dc})}{V_{dc}} = \frac{L}{e_{r_1}} \quad (16)
\]

\[
T_{dc}(V_{dc}) = \frac{L \cdot V_{dc}}{e_{r_1}} \iff i_{s_{ref}(dc)} = \frac{T_{dc}(V_{dc})}{V_{dc}} = \frac{L}{e_{r_1}} \quad (17)
\]

\[
e_{r_1} = \Delta V_{dc}; \text{ Small error caused by variation of DC voltage capacitor.}
\]

![Figure 5: Simplified Backstepping technique scheme](image)

4. Simulation results

![Figure 6: Simulation works in Matlab/Simulink](image)
The Matlab/Simulink simulation tool [12] was used to develop a model that allowed the simulation and testing of the control, which were implemented in the SAPF for three phase four wire systems. The simulation use three phase four wire system 127V 50Hz directly from source as shows in Figure 6.

Figure 7.a: Load currents before harmonics compensation

Figure 7.b: Harmonic source current specter (Phase a).

Figure 8.a: Source currents after harmonics compensation using PI controller

Figure 8.b: Harmonic source current specter (Phase a).

Figure 9.a: Source currents after harmonics compensation using Simplified Backstepping control

4. a. Discussion of results

In Figure 7.a the nonlinear load currents are distorted and unbalanced, while Figures 8.a and 9.a show the source currents after harmonic currents compensation for both controllers PI and simplified Backstepping technique, respectively.

As can be noted, the source currents have almost sinusoidal shapes and the active filter decreases the total harmonic distortion (THD) in the supply currents from...
50.11% to 4.19% (Phase a) with PI controller (Figures 7.b and 8.b). However, with simplified Backstepping controller, the THD is decreased to 2.23% (Phase a) (Figure 9.b) which proves the effectiveness of the Backstepping controller.

According to the simulation wave forms for $V_{dc}$ regulation using PI and simplified Backstepping controllers under nonlinear load after harmonic currents compensation (Figures 10 and 11), we can see that:

The performance of the PI controller suffers from critical overshoots, because it requires precise linear mathematical model, which are difficult to obtain and may not give satisfactory performance under parameter variations, load disturbances…etc. In opposition the simplified Backstepping technique present no overshoots and the response time is the smallest, which demonstrate the effectiveness of this nonlinear controller, especially in case to use when the load is mostly under dynamic regimes.

The simulation results demonstrate that simplified Backstepping control is better than PI control in transient regime; there are a small response time and no overshoot.

As shown in Figures 12, the neutral current is almost cancelled with low ripples.

5. Conclusion

In this paper we have presented a three-phase four-wire SAPF based on the SRF-based algorithms for suppressing all of harmonics current, compensating reactive power and eliminating the neutral current.

The objective was to improve the performance of the three-phase four-wire SAPF a new DC voltage controller, simplified Backstepping technique instead of traditional PI controller.

The simulation results demonstrate that Simplified Backstepping control is better than PI control in transient regime; there are no overshoot and small response time.

Another advantage to implement simplified Backstepping control instead of traditional PI is related to that no precise linear mathematical model of the system is required when Backstepping controller is implemented.

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