

# Optimized Allocation of Phasor Measurement Units in Transmission Systems Using Particle Swarm Optimization

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## I - Introduction

The complexity of the electric power systems (EPS) generated by the interconnection of distant networks, the inclusion of alternative sources and the expansion of the already existing models has hindered the safe operation and monitoring of these networks by their administrators. This challenge comes from the large number of physical points present in the electrical networks that require different monitoring, and the search for ways to minimize the number of meters to reduce costs is crucial, maintaining the possibility of determining the operational state in which a given system. The correct monitoring of a network is important in the identification of faults, in the control of quality of service indicators and in the planning of concessionaires in general. Due to its high data sampling capacity and the possibility of GPS communication, which guarantee numerous benefits in state estimation processes, the use of phasor measurement units (PMUs) has become increasingly common. However, the high cost of these devices requires that their implementation be done in a precise manner to ensure the observability of the network at low cost.

In order to solve this problem in an economically viable way, projects for the distribution of monitors that are low cost and yet guarantee observability and are as redundant as possible, which allows for a better estimation of EPS states.

As it is a problem of a combinatorial nature, the application of meta-heuristics is widely used. In recent years, evolutionary algorithms have been proposed for the design of measurement systems with good results [1,2]. Going a step further, it was demonstrated how these algorithms can be easily adapted to different types of systems. In fact, its practicality would allow the operation in real time to restore observability from pseudo-measurements, with just a few small changes [3,4], in which they use an evolutionary algorithm for state estimation.

Still in the allocation problem, a methodology is presented based on network characteristics with good results in the IEEE test systems and uses an approach based on dynamic programming successfully approximated in large systems [5,6]. Given the flexibility of the meta-heuristics and the history of success in the PMU allocation problem, an algorithm based on particle swarm optimization (PSO) is proposed, capable of performing the allocation with the lowest possible cost, ensuring the observability of the system and seeking greater redundancy. This algorithm was tested with IEEE test systems and a brief statistical analysis and comparison with known results from the literature was made.

## II - Methodology

### A. Modeling the problem

The use of phasor measurement units (PMU) allows the problem to be modeled as a covering problem (PR). Each PMU can provide data related to its installation bar and the currents that are coming out of it, that is, each unit will serve its installation point and everyone directly connected to it. Following the modeling proposed by [6], the problem consists in minimizing equation (1), while meeting the conditions imposed by (2), (3) and (4), being:

$$\min z = \sum_{j=1}^n c(j) * x(j) \quad (1)$$

$$\text{Sujeito a} \quad \sum_{j=1}^n d(i,j) * x(j) \geq b(i) \quad (2)$$

$$0 \leq x(j) \leq 1 \quad (3)$$

$$x(j) \text{ inteiro para } j = 0,1,2, \dots, n \quad (4)$$

In the equations,  $c(j)$  is the cost of a PMU in a bar  $j$ ,  $x(j)$  is a binary variable that says whether there is PMU installed in that bar  $j$  ( $x(j) = 1$ ) or not ( $x(j) = 0$ ), in the total number of bars,  $d(i, j)$  is a binary value that represents the position  $(i, j)$  of the adjacency matrix corresponding to the topology of the analyzed SEP and  $b(i)$  is an element of vector  $B$  that indicates for at least how many PMUs the bar  $j$  must be serviced.

For validation of the method,  $b$  is considered a vector with all elements equal to 1, since it is desired to meet all the bars with at least one meter (minimum cost condition that guarantees the observability of the system), as well as the cost vector it is also considered unitary. The condition of maximizing the sum of the product between the adjacency matrix and the vector of variables is also added, which indicates the total amount of measurements possible to be obtained using all the measurement channels of the system.

## B. Algorithm

A swarm optimization algorithm was implemented based on the proposed model [12]. The evaluation of the solutions or particles is made following the equations presented in the modeling of the problem, considering a hierarchy among the existing objectives. Thus, in the particle evaluation step, it is considered (i) whether the solution has better observability than the compared solution, given by the best location (in this case the value is compared to the product of the value obtained in the solution with the established tolerance) or global, depending on the stage; (ii) if the values in (i) are equal, the costs are compared and the lowest cost is chosen; and (iii) if the values are equal in (i) and equal in (ii), the most redundant solution is chosen.

For the generation of the initial solutions, solutions with small amounts of elements equal to one were used, seeking to accelerate the convergence because it is a problem where there will always be a solution with minimum cost in which the number of installed PMUs will be equal to or less than half the number of buses in the system, where each PMU can serve the installation bar and adjacent buses.

New operators were also created to act in the movement of the particle: tolerance constants in the local evaluation of solutions (in cost and observability), where the best location can change to a worse solution, but close to the optimum location, in order to increase the space search for solutions; and

After the changes, the new algorithm is given as follows:

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Variables
pop, obj, v, tol, it, objL, objG, inertia, better Start
generating pop for i from 1 to it
for j from 1 to size (pop)
obj (j) = evaluate (pop (j)) if obj (j) > olerance * objL (j)
objL (j) = obj (j) if obj (j) > objG
objG = obj (j) best = pop (j) end if
end if
end if
v (j) = rand * (obj (j) - objL (j)) + rand * (obj (j) - objG) +
pop inertia (j) = pop (j) + v (j) end to
order to return objGlobal, better
End
  
```

Pop being the population of particles, obj the performance of each solution for the objectives, v the speed of each particle, tol the search tolerance of the objectives, it the number of iterations, objL the best historical performance of a particle, objG the best performance general history, inertia caused by the movement of the previous iteration and better the best particle found.

## IV - Results

First, we have the results already known from the literature to take as a basis. Figura 1 shows IEEE-14 Buses e Table 2 shows the relationship of the systems with their minimum quantities of PMUs required, reference values for the problem.

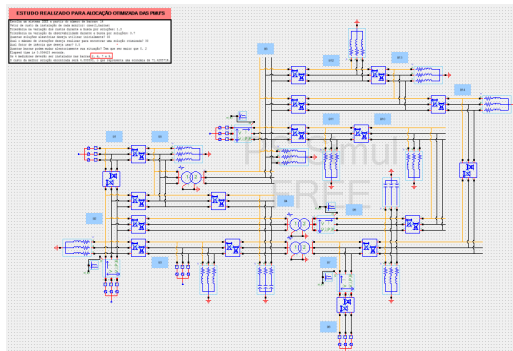


Fig. 1. IEEE 14 Buses

Input System	Minimum PMUs values
14	4
10	10
57	17
118	32

Table2. - Reference values

## 2.1 Performance considering uniform costs

Assuming that PMU installation costs are equal at any point is useful to demonstrate the performance of the algorithm as there will be no considerable difference that would make the algorithm converge to a minimum point. The cost per PMU of 1 unit of cost (u.c.) is then assumed.

In this situation, at least one of the combinations of input parameters was able to find solutions with the quantities of reference PMUs in each case at least once. Only conditions 1 and 3 for the 118 bus system, which did not meet the expected value. In all cases, the solutions ensured total observability of the system.

The best results found, in terms of the number of PMUs for each entry and systems are shown in Table 3.

Despite having the minimum quantities of PMUs equal, the bars with installed units do not coincide in all cases with what is observed in the literature. This does not mean, however, that they are worse solutions, since there may be more than one combination with the same number of measurements possible, the same cost, but different installation positions. This fact occurs in the systems of 30, 57 and 118 buses, and in the cases of 30 and 57 gains are observed in relation to what some meta-heuristic risks or that do not consider the quantity of measurements in the modeling of the problem found.

Input System	1	2	3
14	4 PMUs	4 PMUs	4 PMUs
10	10 PMUs	10 PMUs	10 PMUs
57	17 PMUs	17 PMUs	17 PMUs
118	33 PMUs	32 PMUs	33 PMUs

Table 3. - Best results obtained

Finally, looking at the average performance over the iterations, it is noted that the algorithm finds solutions with total observability in the first iterations and, even the 57-bus system, the cost and quantity of measurements also tend to stabilize quickly. Figures 1, 2, 3 and 4 show the behavior of the algorithm for the combination of input parameters 2 (300 iterations and up to 4 random changes of speed) in the systems of 14, 30, 57 and 118 buses, respectively. In addition, on average, in less than 20 iterations solutions are already found with the observable system and close to the optimal cost. On the one hand, this shows the great efficiency of the PSO in finding solutions quickly, but it indicates the need to create routines that prevent the problem from getting stuck in these minimum points and spend less simulations to find global optimum.

## VI - Conclusions

The results for the IEEE test systems of 30, 57 and 118 buses confirm the efficiency of metaheuristics to solve the problem of allocating PMUs in electrical systems, however it may be necessary to perform several implementations of the algorithms to find truly optimized solutions, already that there is no guarantee of achieving optimum in metaheuristics. It is noted that, even if several executions are necessary, the time spent to find very optimized options of measurement systems is much less than what would be used to test all possibilities or apply some deterministic method.

The way the problem is modeled, also ensures that the algorithms developed and validated under uniform cost conditions and with a minimum requirement of only one measurement per bar can work with varied costs and impose a minimum local redundancy different from one, in addition to allowing work with varied topologies of electrical systems.

Regarding the proposed methodology, despite the good results, optimizations are necessary to prevent the PSO from being trapped in local minimums, especially in larger systems so that it is not necessary to have such a high number of simulations to find optimal solutions. There is also the need to establish a stopping criterion that is not based only on the number of iterations in real applications. A better adjustment of the input parameters is also recommended, since only three different combinations were chosen for comparison purposes only, but in practice, this affects the performance of the algorithm.