Analysis and Comparison Between Different Methods of Current Reference Generation for Active Filters Control

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Abstract. *In modern distribution systems the proliferation of non-linear loads results in a deterioration of the quality of voltage waveforms at the point of common coupling (PCC) of various consumers. Therefore, power-conditioning equipment is becoming more important for electric utilities and their customers. With the rapid development of semiconductor devices in power and control circuits, a new generation of equipment for power quality, the active power filters, has been developed. Their advantages over conventional means are more flexibility and very fast control response.*

The control of an active filter comprises two major parts: the reference current computation and the current control. There are two fundamental methods of generating the reference current: (i) frequency–domain methods, based on the Fourier analysis and (ii) time-domain analyze, based on the theory of instantaneous imaginary power in the three-phase circuits, often called p-q theory.

The paper begins by presenting the principle of active filtering and the basic instantaneous imaginary power theory. In the hypothesis of a distorted and/or unsymmetrical load voltage system, the p-q theory has proven limitations; consequently the paper reviews and evaluates other two reference current calculation methods. Finally a comparative analyze of the three methods features is carried out.

Key words

Active filters, current reference, instantaneous imaginary power, compensation, reactive power

1. Introduction

In recent years, the electrical power quality is a more and more discussed issue. The main problems are stationary and transient distortions in the line voltage such as harmonics, flicker, swells, sags and voltage asymmetries. With the significant development of power electronics technology, especially static power converters (well known as non-linear loads), voltage harmonics resulting from current harmonics produced by the non-linear loads have become a serious problem. Paradoxically, static power converters, the source of most of the perturbations, could also be used efficiently as active power filters in order to cancel or mitigate most of the above mentioned power quality problems as well as other power system problems such as damping of voltage oscillations.

A. Active filters

Active filters are fundamentally static power converters configured to synthesize a current or voltage source. Since their basic compensation principles were proposed around 1970, active filters have been successfully used in harmonic filtering and power factor compensation but also to perform complex tasks in the context of total power quality management. Advantages of active filters over conventional means include: very fast control response, more flexibility in defining and implementing control functions (more than one function can be performed), and no additional resonance introduced into the ac supply.

B. Active Filters Topologies

The most used system configurations of active power filters are illustrated in Figures 1 and 2. Other topologies result from the combination of the two structures and/or with passive filters.

There are two types of power circuits used to implement an active filter, namely voltage source converters and current source converters. With higher efficiency and lower costs, the voltage source converters are usually preferred.

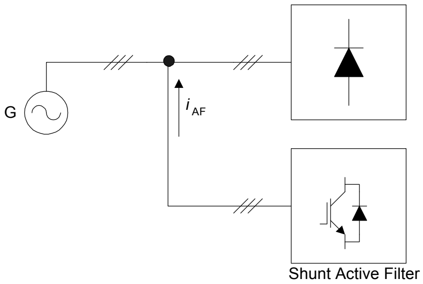

Fig. 1. Shunt active filter used alone.

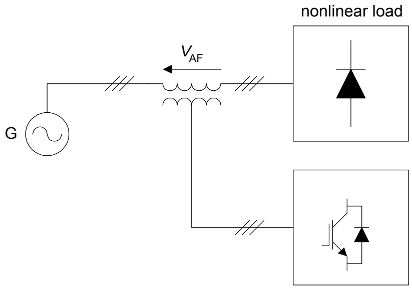

Fig. 2. Series active filter used alone.

2. Principle of active power filtering

Figure 2 illustrates the basic system configuration for a shunt active compensation. It includes the power line, the active filter and the nonlinear load.

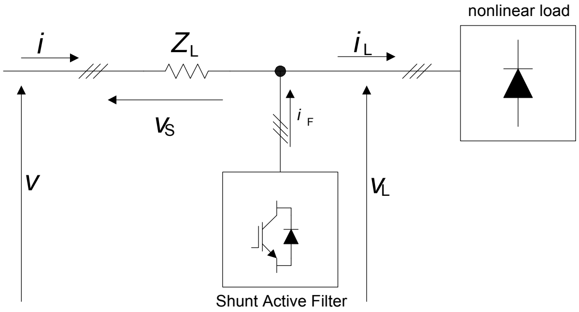

Fig. 2. Principle of active filtering (shunt active filter operation) – block diagram.

The electrical parameters that have to be considered are: source voltage system **v**, line current system **i**, load voltage system $\mathbf{v_L}$, load current system $\mathbf{i_L}$, power line impedance Z_L (that depends on the frequency of the currents **i**), voltage across the power line impedance $\mathbf{v_S}$ and filter current system $\mathbf{i_F}$

$$\mathbf{v}=\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}; \quad i=\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}; \quad \mathbf{v_S}=\begin{bmatrix} v_{s1} \\ v_{s2} \\ v_{s3} \end{bmatrix};$$

$$\mathbf{i_F}=\begin{bmatrix} i_{F1} \\ i_{F2} \\ i_{F3} \end{bmatrix}; \quad \mathbf{i_L}=\begin{bmatrix} i_{L1} \\ i_{L2} \\ i_{L3} \end{bmatrix}; \quad \mathbf{v_S}=\begin{bmatrix} v_{L1} \\ v_{L2} \\ v_{L3} \end{bmatrix}. \tag{1}$$

The current systems can be split in two parts

$$\mathbf{i_X}=\mathbf{i_{Xa}}+\mathbf{i_{Xr}}\,, \tag{2}$$

that is:

- an active component, related with the conventional fundamental active current and the harmonic currents caused by the ac component of the instantaneous real power;
- a reactive component related with reactive power generated by the fundamental components of voltages and currents and the harmonic currents caused by the ac component of instantaneous reactive power.

The active filter must generate a current equal to the reactive component of $\mathbf{i_L}$, $\mathbf{i_{Lr}}$; for each harmonic in $\mathbf{i_L}$ there is a component in $\mathbf{i_{Lr}}$. The value of $\mathbf{i_{Lr}}$ represents the current reference of the active filter and is obtained by measuring the $\mathbf{i_L}$ value and calculating the active component $\mathbf{i_{La}}$. Consequently, the basic relations describing the filter current control is

$$\mathbf{i_{Fa}}=\mathbf{0}\,; \quad \mathbf{i_{Fr}}=\mathbf{i_L}-\mathbf{i_{La}}\,, \tag{3}$$

resulting

$$\mathbf{i_a}=\mathbf{i_{La}}\,; \quad \mathbf{i_r}=\mathbf{0}\,. \tag{4}$$

With such a control algorithm, the line current will include only the active component of the $\mathbf{i_L}$ with regard to $\mathbf{v_L}$.

In order to analyze the system behavior in the presence of harmonics, the following situations have to be considered:

- the voltage system **v** contains harmonics not included in the load current; these harmonics are absent from $\mathbf{v_S}$ but founded again in $\mathbf{v_L}$;
- the current system **i** contains harmonics not included in the source voltage; these harmonics will generate identical frequency components in $\mathbf{v_L}$ and $\mathbf{v_S}$.

It can be also observed that:

- the load voltage system $\mathbf{v_L}$ gathering all the harmonics in **v** and $\mathbf{i_L}$;
- the $\mathbf{i_L}$ active components with frequencies not included in the voltage source spectrum dissipate the energy in the resistive part of line impedance;
- the RMS values of the line current harmonics not included in the load current spectrum are proportional with the RMS values of their reactive components. Consequently, cancellation of the reactive components in **i**, leads to cancellation of the above mentioned line current harmonics.

3. Current Reference Generation

Regarding to the quantity that has to be measured and analyzed in order to generate the current reference signal of the (shunt) active filter control system, there are three kinds of strategies:

- load current detection;
- supply current detection;
- voltage detection.

Load current detection and supply current detection are recommended for shunt active filters working locally, for individual non-linear high-power consumers. Voltage detection is suggested for: (a) shunt active filters functioning in complex equipments (so called “unified power quality conditioner”), whose destination is to equip the primary distribution substations; (b) shunt active filters located in the distribution system and supported by utilities. Also the series active filters are mostly based on supply current detection.

A. Harmonic Detection

There are mainly two kinds of control strategies for analyzing and extracting current or voltage harmonics from the distorted waveforms:

- frequency–domain, based on the Fourier analysis in the frequency-domain;
- time-domain, based on the theory of instantaneous reactive power in the three-phase circuits and often called p-q theory.

B. Instantaneous imaginary power - basic theory

As it was previously shown, one alternative to determine the current reference required by a shunt active filter based on a voltage source inverter is the instantaneous reactive power theory, proposed by Akagi ([1],[4],[5]). The concept is very popular and useful for this sort of applications and it is often used in the assumptions that:

- the source voltage system is sinusoidal and satisfies the condition

$$v_1 + v_2 + v_3 = 0\,; \tag{5}$$

- the harmonic components in the load voltage system arise from the line currents flowing in the line impedances.

Initially, the method requires a transformation of the voltage and current signals, from classical *[a b c]* frame in α–β plane (E. Clarke components). The instantaneous values of voltages and currents in the α–β coordinates are obtained as following

$$\begin{bmatrix} v_{La} \\ v_{Lb} \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}; \quad \begin{bmatrix} i_{La} \\ i_{Lb} \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} i_{L1} \\ i_{L2} \\ i_{L3} \end{bmatrix}, \tag{6}$$

where **A** is the transformation matrix

$$A = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}. \tag{7}$$

The instantaneous active and reactive power absorbed by the load can be expressed, in α–β coordinates, as

$$p(t) = v_{La}(t) \cdot i_{La}(t) + v_{Lb}(t) \cdot i_{Lb}(t)\,, \tag{8}$$

$$q(t) = v_{La}(t) \cdot i_{Lb}(t) - v_{Lb}(t) \cdot i_{La}(t)\,, \tag{9}$$

where *p* corresponding to the conventional instantaneous real power defined in *[a, b, c]* reference frame, and *q* is a new electrical quantity ([1],[4],[5]) defined as instantaneous imaginary power, which is represented by the product of the instantaneous voltage and current, but can not be treated conventionally.

Relationships (8) and (9) can be written in a matrix form as

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_{La} & v_{Lb} \\ -v_{Lb} & v_{La} \end{bmatrix} \cdot \begin{bmatrix} i_{La} \\ i_{Lb} \end{bmatrix}. \tag{10}$$

Consequently, the $\mathbf{i}_\mathrm{L}$ current components in α–β plane, as a function of the instantaneous power, are given by

$$\begin{bmatrix} i_{La} \\ i_{Lb} \end{bmatrix} = \begin{bmatrix} v_{La} & v_{Lb} \\ -v_{Lb} & v_{La} \end{bmatrix}^{-1} \cdot \begin{bmatrix} p \\ q \end{bmatrix} = \\ = \frac{1}{\Delta} \cdot \begin{bmatrix} v_{La} & -v_{Lb} \\ v_{Lb} & v_{La} \end{bmatrix} \cdot \begin{bmatrix} p \\ q \end{bmatrix}\,, \tag{11}$$

where

$$\Delta = \left(v_{La}^2 + v_{Lb}^2 \right) \tag{12}$$

is called voltage norm.

The values of *p* and *q* in (11) can be expressed in terms of the dc component plus the ac components,

$$p = \bar{p} + \tilde{p} \\ q = \bar{q} + \tilde{q} \tag{13}$$

where:

- $\bar{p}$ is related with the conventional active power;
- $\tilde{p}$ is related with the active power caused by the harmonic currents;
- $\bar{q}$ is related with the reactive power generated by the fundamental components of voltage and currents;
- $\tilde{q}$ is related with the reactive power caused by the harmonic currents.

In order to compensate reactive power and current harmonics generated by non-linear loads, the reference signal of the shunt active power filter must include the values of $\tilde{p}$, $\bar{q}$ and $\tilde{q}$, and can be calculated with the following expression:

$$\begin{bmatrix} i_{Fa} \\ i_{Fb} \end{bmatrix} = \begin{bmatrix} i_{La} \\ i_{Lb} \end{bmatrix} - \begin{bmatrix} i^*_{La\_a} \\ i^*_{La\_b} \end{bmatrix} = \frac{1}{\Delta} \cdot \begin{bmatrix} v_{La} & -v_{Lb} \\ v_{Lb} & v_{La} \end{bmatrix} \cdot \begin{bmatrix} \tilde{p} \\ \bar{q} + \tilde{q} \end{bmatrix}, \tag{14}$$

where

$$\begin{bmatrix} i^*_{La\_a} \\ i^*_{La\_b} \end{bmatrix} = \frac{1}{\Delta} \cdot \begin{bmatrix} v_{La} & -v_{Lb} \\ v_{Lb} & v_{La} \end{bmatrix} \cdot \begin{bmatrix} \bar{p} \\ 0 \end{bmatrix} \tag{15}$$

With (14), the final compensating currents, including the zero sequence components in *[a b c]* reference frame are given by

$$\mathbf{i_F} = \begin{bmatrix} i_{F1} \\ i_{F2} \\ i_{F3} \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} -i_0 \\ i_{Fa} \\ i_{Fb} \end{bmatrix}, \qquad (16)$$

where

$$i_0 = \frac{1}{\sqrt{3}} \cdot \left(i_{L1} + i_{L2} + i_{L3}\right). \qquad (17)$$

The main difficulty in current reference generation for active filters control consists in identification of the load/supply active current component. The instantaneous reactive power theory replaces this component with the values given by (15); therefore, as long as *q* is only the instantaneous imaginary power, the $\mathbf{i^*_{La}}$ current system is not necessarily representing the load active current component, as it was considered in (2).

C. Limitations of the method

Different theoretical and experimental approaches ([2],[3]) revealed that the instantaneous imaginary power theory does not provide good performances dealing with distorted voltage and/or unsymmetrical systems and consequently, extensions of the basic method have been developed in order to overtake the above mentioned constraints.

The shunt active filter currents are expressed in *[a b c]* reference system by

$$\mathbf{i_F} = \mathbf{i_L} - \mathbf{i^*_{La}} \qquad (18)$$

where $\mathbf{i^*_{La}}|_{[a\ b\ c]}$ currents result from $\mathbf{i^*_{La}}|_{\alpha-\beta}$ (15), applying the inverse Park transformation.

By accepted definition, the current active components must have the same frequency and phase as the voltage components. Consequently, if the $\mathbf{i^*_{La}}|_{\alpha-\beta}$ currents represent the active components of $\mathbf{i_L}$, their values must be proportional with those of $\mathbf{v_L}|_{\alpha-\beta}$, condition achieved if Δ is a constant (time invariant quantity).

This condition is attended in some cases, but is not the general case. Therefore, this form of applying the method can introduce notably errors.

4. Modified instantaneous imaginary power methods

Consequently, the paper reviews and evaluates other two reference current calculation methods.

A. Method 1

In [2], the authors propose a modified method that avoids the above mentioned inconvenient. In the three-phase systems, currents $\mathbf{i^*_{La}}|_{\alpha-\beta}$ become

$$i^*_{Lj} = \frac{v_{Lj} \cdot \bar{p}}{\left(v^2_{L1} + v^2_{L2} + v^2_{L3}\right)}. \qquad (19)$$

By defining a time-independent real conductance *g*, such that

$$i_{Laj} = v_{Lj} \cdot g\,|_{j=1\ldots3}\,, \qquad (20)$$

it is obtained an equivalent current system, $\mathbf{i_{La}}$,

$$\begin{aligned} \mathbf{i_{La}} &= i_{Laj}\,|_{j=1\ldots3} = \frac{v_{Lj} \cdot \bar{p}_{\mathrm{mod}}}{mean\,(v^2_{L1} + v^2_{L2} + v^2_{L3})}\,|_{j=1\ldots3}\,, \\ \bar{p}_{\mathrm{mod}} &= \bar{p} \cdot \frac{1}{2p} \int_0^{2p} \frac{v^2_{L1} + v^2_{L2} + v^2_{L3}}{k_v}\,dq \end{aligned} \qquad (21)$$

corresponding to the same active power as currents $\mathbf{i_L}$. These currents are permanently proportional and in phase with the corresponding voltages and the voltage harmonics are all duplicated in the current active components.

B. Method 2 (extension of p-q theory)

This algorithm was developed [3] in order to avoid the limitations of the initial instantaneous imaginary power theory. The method considers the *p* and *q* defined as

$$p = v_{L1} \cdot i_{L1} + v_{L2} \cdot i_{L2} + v_{L3} \cdot i_{L3}\,; \qquad (22)$$

$$q = v'_{L1} \cdot i_{L1} + v'_{L2} \cdot i_{L2} + v'_{L3} \cdot i_{L3}\,; \qquad (23)$$

where v_{L1}', v_{L2}', v_{L3}' lag v_{L1}, v_{L2} and v_{L3} by 90^o.

Kirchhoff's current law for a three-phase three-wire system gives

$$i_{L1} + i_{L2} + i_{L3} = 0\,. \qquad (24)$$

Combining (22)-(24) results

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_{L1} - v_{L3} & v_{L2} - v_{L3} \\ v'_{L1} - v'_{L3} & v'_{L2} - v'_{L3} \end{bmatrix} \cdot \begin{bmatrix} i_{La} \\ i_{Lb} \end{bmatrix}, \qquad (25)$$

$$\begin{bmatrix} i_{La} \\ i_{Lb} \end{bmatrix} = \frac{1}{\Delta} \cdot \begin{bmatrix} v'_{L2} - v'_{L3} & v_{L3} - v_{L2} \\ v'_{L3} - v'_{L1} & v_{L1} - v_{L3} \end{bmatrix} \cdot \begin{bmatrix} p \\ q \end{bmatrix}, \qquad (26)$$

where

$$\Delta = \left(v_{L1} - v_{L3}\right) \cdot \left(v'_{L2} - v'_{L3}\right) - \left(v'_{L1} - v'_{L3}\right) \cdot \left(v_{L2} - v_{L3}\right). \qquad (27)$$

According to the instantaneous imaginary power theory, the current reference of the active filter is

$$\begin{bmatrix} i_{Fa} \\ i_{Fb} \end{bmatrix} = \begin{bmatrix} i_{La} \\ i_{Lb} \end{bmatrix} - \begin{bmatrix} i_{La\_a} \\ i_{La\_b} \end{bmatrix}, \qquad (28)$$

where i_{La_a} and i_{La_b} are ideal source current (responsible for the active power consumption of the load)

$$\begin{bmatrix} i_{La\_a} \\ i_{La\_b} \end{bmatrix} = \frac{1}{\Delta} \cdot \begin{bmatrix} v'_{L2} - v'_{L3} & -\left(v_{L2} - v_{L3}\right) \\ -\left(v'_{L1} - v'_{L3}\right) & v_{L1} - v_{L3} \end{bmatrix} \cdot \begin{bmatrix} \bar{p} \\ 0 \end{bmatrix} = \qquad (29)$$

$$= \frac{\bar{p}}{\Delta} \cdot \begin{bmatrix} v'_{L2} - v'_{L3} \\ -\left(v'_{L1} - v'_{L3c}\right) \end{bmatrix}$$

while $\bar{p}$ is the mean value of p.

5. Comparative evaluation of the p-q based theory methods

The instantaneous imaginary power theory is widespread in calculation of reference current required by the control systems of active power filters. The method can successfully work in applications requiring harmonics cancellation and/or reactive power compensation.

Basic p-q theory ([1],[4],[5]) has proven to be inaccurately ([2],[3]) when the load voltage system is distorted and/or unsymmetrical. In order to compensate the limitations, the method has been improved and extended.

In [3], the authors expose a reliable detection method of load active current components, in the assumption of a distorted load voltage system. Examples have proven that applying the p-q theory in such a system will lead to false active components in the $\mathbf{i}_{\mathbf{La}}$ currents system, while the modified method 1 shows a reduction of fictitious load current harmonics.

The examples and experiments ([3]) with the extension of the p-q method (modify method 2) shows that:

- in unsymmetrical voltage systems, the load active current estimated by the p-q theory is distorted, while that calculated by the extension p-q theory has no distortion;
- in unsymmetrical voltage systems, the value of the voltage norm Δ is almost constant with the extension of p-q theory, while in case of basic p-q method, it contains harmonics, being a source of errors;
- extension of p-q theory works satisfactorily with unsymmetrical and distorted load voltage systems, but induces unacceptable errors in symmetrical distorted voltage systems;
- in both above revealed situations, the classic instantaneous imaginary power theory does not work properly.

6. Conclusions

With the rapid development of semiconductor devices in power and control circuits, a new generation of equipment for power quality, namely the active power filters, has been developed. Its advantages, over conventional means, are more flexibility and very fast control response.

The control of an active filter comprises two major parts: the reference current computation and the current control. There are two fundamental ways of generating the reference current: applying frequency–domain methods, based on the Fourier analysis, and time-domain analyze, based on the theory of instantaneous imaginary power in the three-phase circuits, often called as p-q theory.

The paper starts by presenting the principle of the active filtering and the basic instantaneous imaginary power theory. As, in the hypothesis of a distorted and/or unsymmetrical load voltage system, the p-q theory has proven limitations, the paper reviews and evaluates other two reference current calculation methods. A comparative analyze of the three methods features is finally presented.

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