

# Variable Speed Drive modelling of Wind Turbine Permanent Magnet Synchronous Generator

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**Abstract.** With today's possibilities for signal processing, controlled drives with permanent magnet synchronous machines are in attractive solution of several applications and offer many promises. In this paper, the operation of variable speed, stall regulated wind turbine PM synchronous generator, is examined. The analysis and design of control system for an electric drive calls for a dynamic model of the machine. Analysis of the electric part of the system, i.e. the electric generator, the power electronics converters and the control, is carried out. The emphasis is placed on the investigation of a series of computer simulations and the speed control system is designed to maximise the power output and achieve a smooth torque and power profile.

## Key words

Wind turbine generator, PM synchronous machines, electrical drive, variable speed control

## 1. Introduction

The speed of synchronous generators with constant rotor excitation is determined by the stator frequency and the number of poles. As long as no efficient, variable frequency power supply was available this meant constant speed operation and fixed frequency. There are drives applications, where constant speed is desired or where the reactive power that can be generated is an important feature. Another field of application where constant speed is not feasible, is wind turbine synchronous generators with permanent magnet excitation and electronic converter. Their main advantage, when compared with induction machines, is the elimination of rotor losses; on the other hand, field weakening is more difficult [5,6].; with a synchronous machine this may be of considerable complexity, if details are to be taken into account. However, considerable simplification may be gained without much loss of information, by restricting ourselves to realistic assumptions such as neglecting saliency and damping effect, ... The design of a wind turbine system is a complicated task due to the high number of parameters involved and the often conflicting requirements, such as the low cost and raggedness on the one hand, and the

good output power quality and satisfactory dynamic characteristics on the other. In this paper, we consider a permanent magnet synchronous generator equipping a stall regulated variable speed wind turbine. In order to achieve variable speed operation, a power electronics converter is used interfacing the generator to the network. This converter consists on a uncontrolled series bridge 3-phase diode rectifier, a DC-DC converter, a 3-phase PWM voltage source inverter. A control loop of the speed and the torque is also added. The measured rotor speed determines the torque reference, which is used to calculate the DC current reference value. This value is tracked by a hysteresis control loop of the DC-DC converter. The output inverter regulates the DC voltage and its rated value, as well as the output power factor. The modelling method is carried out both for the study state and dynamic conditions. The effectiveness of the proposed control system should be then confirmed by the parameter values.

Fig. 1 shows the structure of the studied system; possibly, a step-up transformer between the generation system and the grid, may be added.

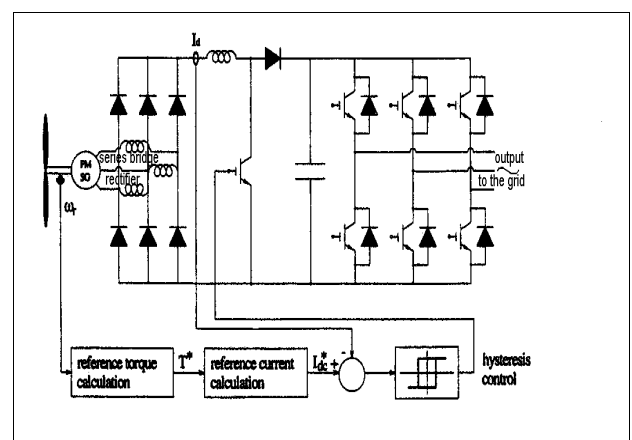


Fig. 1 Description scheme of the electrical power generation system of the wind turbine.

## 2. Modelling Method

## A. Mechanical Part Study

### 1) Stochastic wind speed model

Studies were already carried out to simulate numerically the wind speed which is considered as a random process. This process can be assumed to two components [2], [3]:

- the slower component, which describes the slow evolution of the wind in a defined time horizon.
- the turbulent component, considered as non-stationary, is assumed to be dependent on the slower one.

In the wind spectral characteristic modelling method of Van Der Hoven, the turbulence component is considered as a stationary random process where fluctuations magnitude does not depend on the average. Wind speed is obtained by means of discretization on the power spectral characteristic  $S_w$ , as follows:

- Firstly, discretization of pulsations  $w_i$ ,
- Then, calculation of the areas between the  $S_w(w_i)$  curve and pulsation  $w_i$ , which correspond to consecutive discrete values of the pulsation.

$$S_i = \frac{1}{2} [S_w(w_i) + S_w(w_{i+1})] (w_{i+1} - w_i) \quad (1)$$

- The magnitude  $A_i$  of each spectral component corresponding to  $w_i$ , is given by (2):

$$A_i = \frac{2}{\pi} \sqrt{S_i} \quad (2)$$

- The wind speed is the sum of harmonics characterised by  $A_i$ ,  $w_i$ , and phase  $\varphi_i$ , generated in a way random. The model should have more actual signification with a non stationary component of the wind speed as follows:

$$v_w(t) = \frac{2}{\pi} \sum_{i=0}^{N_1} A_i \cos(w_i t + \varphi_i) + \frac{2}{\pi} \sum_{i=0}^{N_2} A_i \cos(w_i t + \varphi_i) \quad (3)$$

$N_1$ : samples for the slow component (first term of eq.(3))  
 $N_2$ : samples for turbulence component (second term) which amplitude is adjusted by a coefficient  $K$  with a filter [2],[3], which time constant depends on the direct component. Fig. 2 illustrates his method with a sampling period  $T_e = 1$ sec., on a temporal range of 5 hours [3], [4].

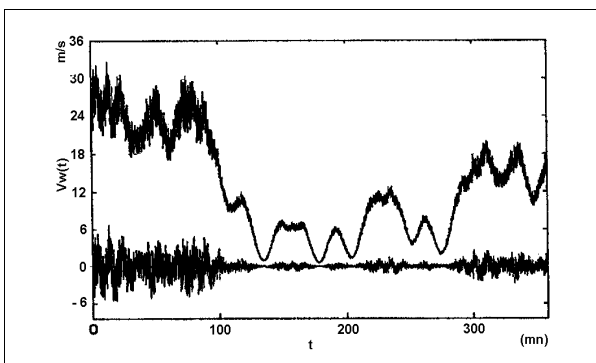


Fig. 2 Generated non-stationary wind speed as function of time

### 2) Aerodynamical wind turbine model

The blades are considered rigidly attached to the WT, consequently the pitch angle of the blades is constant. Taking into account the mechanical and aerodynamical parameters as follows:

$\rho = 1.25 \text{ kg} / \text{m}^3$  is the air density;  $A = \pi R^2$  is the rotor surface;  $V_w$  is the wind velocity and  $C_p(\lambda)$  is the

aerodynamic power coefficient as function of the tip speed ratio  $\lambda = R\omega_m / V_w$ . The mean mechanical input power  $P_m$  at the generator shaft can be given from formula (4):

$$P_m = \frac{1}{2} \rho A V_w^3 C(\lambda) \quad (4)$$

For a given  $C_p(\lambda)$  [2], the mechanical power can be plotted as function of the rotor speed for different values of the wind speed as shown by Fig. 3 issued from Betz's characteristics (parametered by the wind speed; the dashed line indicates maximums of generated power) [1].

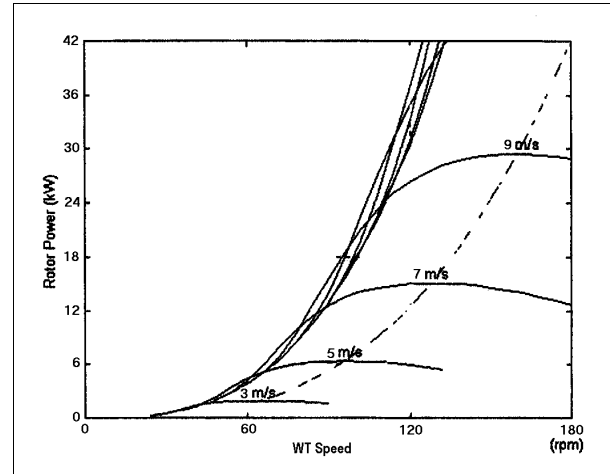


Fig.3 Mechanical power on the generator shaft as function of the WT velocity issued from Betz's characteristics

The rotor aerodynamic torque harmonics must be taken into account, since they affect the output power quality. They are modelled as variation of the wind speed acting at the aerodynamic center of each blade [1]. For a 3-blade wind turbine, the distance to the ground of each blade is function of its angular position  $\theta_i$ , is given by formula (5), where  $H_{hub}$  is the hub height and  $c < l$  defines the distance of the aerodynamic center of the blade from its root:

$$H_i = H_{hub} + cR \cdot \sin\theta_i \quad (5)$$

The variation of wind speed with height is then given by equation (6) per blade:

$$\frac{V_{w,i}(h_i)}{V_w(H_{hub})} = \left( \frac{h_i}{H_{hub}} \right)^a \quad (6)$$

Where  $a = 0.17$  typically. The tower shadow can practically be described as a sinusoidal as a sinusoidal reduction of wind speed as the blade passes in front of the tower, wind flow is obstructed. These effects are modelled by expression (7):

$$V_w^{sh}(\theta, t) = V_w(t) \left( \frac{H_{hub} + cR \sin\theta}{H_{hub}} \right)^a f(\theta) \quad (7)$$

where  $f(\theta) = 1 - \Delta V_w \sin \left[ \frac{\pi}{2\theta_{sh}} \left( \theta - \left( \frac{3\pi}{2} - 9_{sh} \right) \right) \right]$

when:  $\frac{3\pi}{2} - \theta_{sh} + 2k\pi \leq \theta \leq \frac{3\pi}{2} + \theta_{sh} + 2k\pi$  ; and  $f(\theta) = 1$

elsewhere.  $\Delta V_w$  is the maximum wind speed drop, and  $\theta_{sh}$  is the effective tower shadow angle.

Hence, the total mechanical power available at the generator shaft is given below by expression (8), [8]:

$$P_m = \frac{1}{2} \rho A \sum_1^3 [V_w^{sh}(t, \theta_i)]^3 \left\{ \frac{1}{3} C_p(\lambda_i) \right\} \quad (8)$$

### B. Study of the PM Synchronous Generator and power Converters

#### 1) Alternator Park model

For this purpose, we use the machine equations issued from Park's transform q/d axes [8]:

$$v_{qs} = -r_s i_{qs} + \frac{d\lambda_{qs}}{dt} + \omega_r \lambda_{ds} \quad (9)$$

$$v_{ds} = -r_s i_{ds} + \frac{d\lambda_{ds}}{dt} - \omega_r \lambda_{qs} \quad (10)$$

$$v_{os} = -r_s i_{os} + \frac{d\lambda_{os}}{dt} \quad (11)$$

$$\lambda_{qs} = -L_{ls} i_{qs} - L_{mq} I_m \quad (12)$$

$$\lambda_{ds} = -L_{ls} i_{ds} + L_{md} (-i_{ds} + i_m) \quad (13)$$

$$\lambda_{os} = -L_{ls} i_{os} \quad (14)$$

Where  $v_{qs}$ ,  $i_{qs}$ ,  $\lambda_{qs}$ ,  $v_{ds}$ ,  $i_{ds}$ ,  $\lambda_{ds}$ ,  $v_{os}$ ,  $i_{os}$ ,  $\lambda_{os}$ , are respectively, q, d, and o axis voltage, current and flux linkages;  $i_m$  is the equivalent magnetising current of the permanent magnets,  $r_s$ ,  $L_{ls}$ , are the armature winding resistance and leakage inductance, and  $L_{mq}$ ,  $L_{md}$ , are the q and d axis magnetizing inductances. The electrical angular speed  $\omega_r$  is calculated by (15) where  $J$  is the inertia moment,  $p$  is pole pairs,  $T_m$   $T_e$  are the mechanical and electromagnetic torques (16).

$$T_m - T_e = J \frac{2}{p} \frac{d\omega_r}{dt} \quad (15)$$

$$T_e = \frac{3}{2} \frac{P}{2} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \quad (16)$$

#### 2) Power electronics converters modelling

The 3-phase output of the PMSG is rectified with a full wave diode bridge rectifier, filtered to remove significant ripple voltage components, and fed a DC-DC boost converter. For an ideal (unloaded and loss-less) PMSG, the line to line voltage is given as [5],[6]:

$$V_{ll}(t) = K_{vl} \omega_e \sin(\omega_e t) \quad (17)$$

where  $K_{vl}$  is the voltage constant in V/rad/s and  $\omega_e = p \cdot \omega_r$ , is the electrical frequency related to the mechanical speed  $\omega_r$ , and  $p$  is number of poles.

The DC rectifier voltage is given by (18), with  $I_{dc}$ , the average rectified PMSG current and  $L_s$  the stator inductance. If we neglect losses, the electrical power output is given by (19).

$$V_{dc} = 3 \frac{\sqrt{2}}{\pi} V_{ll} - \frac{3}{\pi} \omega_e L_s I_{dc} \quad (18)$$

$$P_{dc} = V_{dc} I_{dc} = K_e \omega_r I_{dc} - K_x \omega_r I_{dc}^2 = V_{dc} \frac{K_e}{K_x} - \frac{V_{dc}^2}{K_x \omega_r} \quad (19)$$

where:

$$K_e = \frac{3pK_{vl}}{\pi}; \quad K_x = \frac{3pL_s}{\pi}$$

The models of both rectifier and boost converter, has been developed and used in the simulations. Switching operation of the PWM inverter has been considered as fast enough to maintain the DC link voltage at its reference value, and consider its operation does not affect the WT dynamics [1].

### 3. Generator Operation at steady state

Only fundamentals of voltages and currents are considered to be taken into account. Low frequencies of the rectifier and high ones of the booster are neglected as well as the commutation phenomena in the diodes. Thus, the generator power factor is equal to 1 and the reactive power is equal to zero ( $Q = 0$ ), and by using the Park equations in the above, we obtain the following relation (20):

$$L_q I_{ds}^2 + L_d I_{ds}^2 - L_{md} I_m I_{ds} = 0 \quad (20)$$

Hence, points which co-ordinates are d and q components of the current describe an elliptical motion as given by (20). Note the no dependence of rotor speed. Besides this equation introduces an upper limit to the electromagnetic torque of the generator. Expression (21) gives the maximum of this torque, neglecting the saliency; we could notice that leakage inductance reduce the maximum torque. Equation (22) defines a sort of base torque according to which, the maximum torque could be written more significantly by (23), [8], in which the synchronous reactance appears as determinant for steady state operation. Its value must preliminarily be determined, especially in case of surface mounted magnets [1][8].

$$T_{e,max} = \frac{1}{2} \frac{3}{2} \frac{P}{2} \frac{L_{md}^2}{L_d} I_m^2 \quad (21)$$

$$T_b = \frac{S_b}{\omega_{mb}} \quad (22)$$

$$T_{e,max} = \frac{1}{2} \frac{T_b}{X_d} \quad (23)$$

### 4. Power Conditioner Modelling

#### A. Control method

In order to ensure the feeding of the DC variable load, without over current default, a power conditioning structure based on a dc-dc converter with a feedback

control of outputs voltage and current, has been developed. For the purpose of modelling, a constant dc voltage is assumed; this condition could be obtained with a specific voltage strategy control [7]. A loop of voltage proportional regulator is attached to the wind generator. In this case, the voltage current characteristic of the loop has a slope depending on the gain value of the controller. The reference value of the voltage is provided by the speed regulator. The speed loop ensures an optimal conversion of wind power. The wind generator provides under a given wind regime the correspondent current to the maximal power, delivered by the optimal tip speed ratio. If the wind speed increases, the speed loop modifies the reference value of the proportional voltage regulator. From the power curves shown by in Fig. 2, turbine torque characteristics, are issued with maximum wind energy extraction condition determination, [1], [8], [9], Fig. 4. However, the turbine must be protected from over power conditions due to sudden wind gusts. For this purpose a limit of rotor speed is fixed. In this way, blade stall effect constitutes a sort of advantage for limiting wind overshoots, since the energy loss due to that effect is not really significant.

The torque – speed characteristic is implemented in the reference torque calculation. The torque speed set point is provided from measure; it is then regulated by current controller. The control system has both tasks of maximising energy and filtering the fast variations of the input power, due to wind fluctuations and torque harmonics.

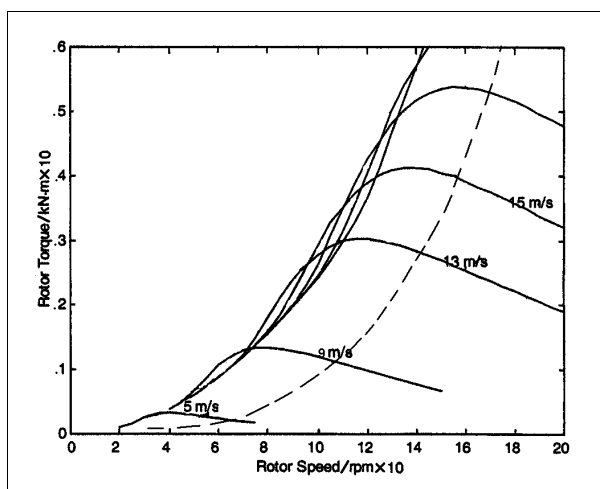


Fig. 4 WT torque as function as rotor speed velocity issued from Betz's characteristics

### B. Linearized Model

An additional low pass filter has been included in the rotor speed feed back. Its task is to attenuate speed oscillations, which could affect generator torque altering the output power quality and the mechanical torques. Transfer function of such a filter is given by (24), [1], [8], where  $\omega_o$  is the initial operating point about which linearization is made;  $T_{eo}$  is the initial torque and  $a_o$  is the correspondent linearization coefficient;  $T_f$  is the filter time constant, and  $J$  is the total inertia of the turbine.

$$\Delta P_e(s) = \frac{\omega_o a_o + T_{eo}}{a_o} \cdot \frac{\left[ \frac{T_f T_{eo}}{\omega_o a_o + T_{eo}} s + 1 \right] \Delta T_m(s)}{\frac{J T_f}{a_o} s^2 + \frac{J}{a_o} s + 1} \quad (24)$$

The low pass filter essentially transforms the system into a second order one. By appropriate selection of the time constant, it is possible to set the poles so that the attenuation is maximized in the cut-off frequency range. The initial operating point is chosen at a high speed, where the system is less damped, and the mechanical stresses as well as the variation of the output power, are more intense [1], [9].

## 5. Simulation results

The complete model of the system has been implemented on matlab –simulink environment for different values of the load. Stochastic wind model has been used for this purpose, Fig. 5.

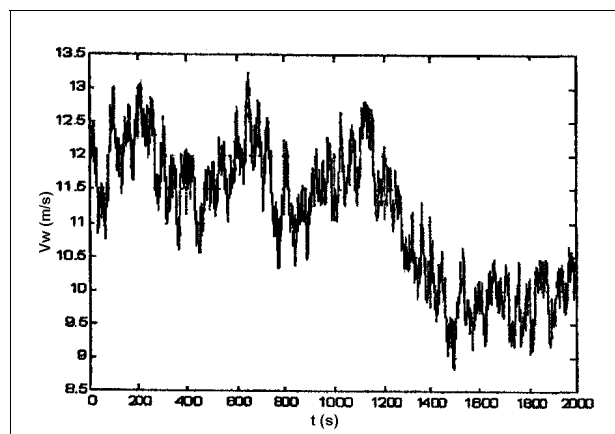


Fig. 5 Stochastic wind speed evolution

The detailed modelling of the generator, the rectifier and dc-dc converter is very important for obtaining the accurate picture of all the system waveforms (voltages, currents, torque, power, speed), this is necessary for dimensioning of semiconductors, and for validating the average value models used in the control system design. Besides, electromagnetic oscillations can rise from generator current harmonics, which could alter the mechanical characteristics of the WECS. Fig. 6 shows the mean generator shaft speed as function of the time.

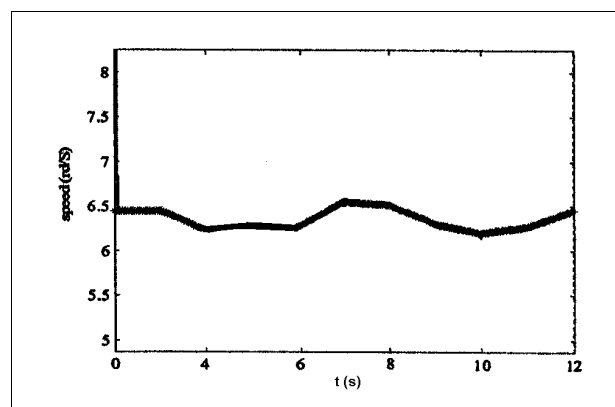


Fig. 6 Generator rotor speed

Concerning the equivalent structure of the power conditioner and the linearization method discussed above, it is more convenient to study its behaviour using Bode diagrams. Fig. 7 and Fig. 8 show respectively, Bode Diagram related to open loop and to feedback control loop; for the first one with open loop, as the poles of the power stage are close to imaginary axis, dynamic behaviour is unsatisfactory. Feedback loop improves system stability and good transient operation. We notice that feedback controller ensures wide frequency bandwidth for output voltage regulation, wide phase range (Fig.8) and fast dynamic response as shown in Fig.9, [2].

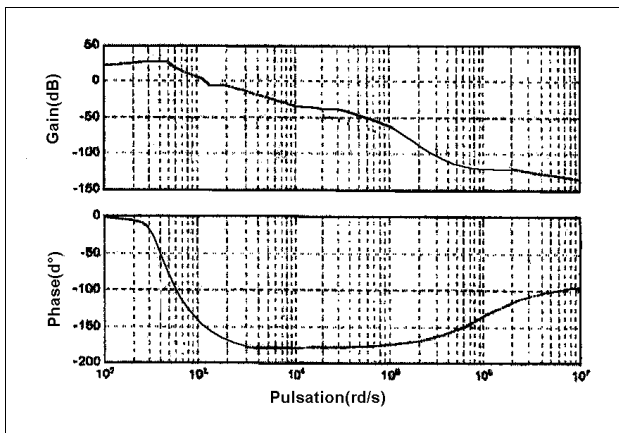


Fig. 7 Bode diagram in open loop case

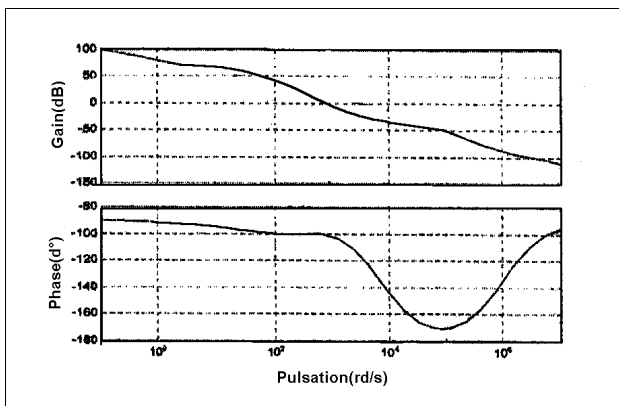


Fig. 8 Bode diagram with feedback control loop

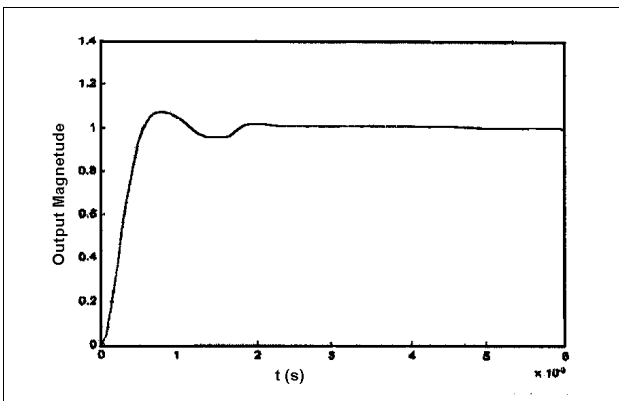


Fig. 9 Step response with feedback control loop

## 6. Conclusion

The present study has been carried out for the matter of modelling a WECS under generated non-stationary wind speed and variable load. Firstly, a modelling method, of both aerodynamical, (issued from Betz's curves) mechanical and electrical aspects, has been developed. The second development focused a specific power conditioner structure; our main goal was to improve the stability and the dynamics of this device. Optimisation of speed control has been performed, obtaining an optimisation of wind energy conversion.

The present study is purely analytical and numerical, inspired from valuable previous studies [1], [2], [7], [8], [9]; its perspectives in the future, should be more practical by implementation of the method on a 1 kW lab – WECS.

## References

- [1] D. C. Aliprantis, S. A. Papathanassiou, M. P. Papadopoulos, A. G. Kaladas, "Modelling and control of a variable speed wind turbine equipped with permanent magnet synchronous generator" in *Proc. of ICEM'2000*, Vol. 3, pp. 558-562.
- [2] M. Elmokadem, C. Nichita, B. Dakyo, W. Koszczara, "Control strategy for a variable load supplied by a wind-diesel system", in *Electromotion journal*, Vol. 10, n° 4, Oct.- Dec.2003, pp. 635-645.
- [3] R. B. Chedid, S. H. Karaki, C. El-chamali, "Adaptive fuzzy control for wind – diesel weak power systems" in *IEEE Trans. on Energy Conversion*, Vol. 15, n°1, March 2000, pp. 71-78.
- [4] C. Nichita, D. Lucas, B. dakyo, E. ceanga, "large band simulation of the wind speed for real time wind turbine simulators", in *IEEE Trans. on Energy Conversion*, Vol. 17, n°4, Dec. 2002, pp.523-530.
- [5] M. J. Ryan, R. D. Lorenz, "A novel controls oriented model of PM generator with diode bridge output" in *Proc. of EPE'97*, Vol. 4, pp. 1.324-1.329.
- [6] M. N. Eskander, "Neural network controller for a PM generator applied in a wind energy conversion system" *Renewable Energy*, vol.26, 2002, pp.463-477.
- [7] M. El Mokadem, "Structure d'un conditionneur de puissance pour un système éolien – diesel" in *Proc. of JCGE'03*, St Nazaire, june 03, pp. 635-640.
- [8] D. C. Aliprantis, "Modeling and control of variable speed wind turbine with a PM synchronous generator" *Thesis report* NTUA, 1999.
- [9] M. Fassenet "Optimal design of dc PM motor intended to an automotive application" *revue générale de Génie Electrique*, Vol. 6 – n° 3-4, Sept. 2003, pp. 351-356.
- [9] S. A. Pa pathanassiou, M. P. Papadopoulos, "Dynamic behavior of variable speed wind turbines under stochastic wind", in *IEEE Trans. on Energy Conversion*, Vol. 14, No 4, Dec. 99.
- [10] A. Cavagnino, M. Lazzari, F. Profumo "A comparison between the axial flux and the radial flux structures for PM synchronous motors", *IEEE Trans. On industrial applications*, Vol. 38, n° 6, Nov. / Dec. 2002, pp. 1517-1524.

- [11] T. Lubin, H. Razik "Magnetic saturation effects on the control of a synchronous PM machine" *IEEE Trans. On Energy Conversion*, Vol. 17, n° 3, Sept. 2002, pp. 356-362.
- [12] M.S. Carnelli, F. Castelli-Dezza, G. Suprti-Furga "Smart modulation: A new approach to power converter control" *EPE 2001 Proceedings*, 27-29 Aug. 2001, Graz, Austria.
- [13] W. Leonhard "Control of Electrical Drives" Ed. Springer Verlag, Berlin 1990.
- [14] W. Leonhard "MP controlled ac servo drives with synchronous machines" *Proc. of IEE conf. On Power Electronics and Variable speed drives*, London 1993, pp.233.
- [15] G. Pfaff "Design and experimental results of brushless ac servo motor" *IEEE industrial Applications annual Meeting*, pp.69