Multivariable QFT controllers design for heat exchangers of solar systems

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Abstract. This paper addresses the control of a heat exchanger placed in a solar water heating system and influenced by external disturbances at plant output. Heat exchangers play an essential role in industries that use renewable sources for energy generation and water heating, i.e. geothermal, solar, ocean, etc. One of the main control targets of such systems is to achieve a simultaneous and accurate control of some temperatures. A multivariable (MIMO) model of the heat exchanger of a solar plant, and a robust controller able to govern the system despite the external disturbances and loop interactions are developed in this work. The MIMO methodology for the non-diagonal controller design is based on the Quantitative Feedback Theory (QFT).

Key words

Multivariable control, QFT control, solar systems, heat exchanger, power plants, disturbance rejection.

1. Introduction

In a society with increasing energy demand and decreasing supplies it is necessary to develop the potential renewable resources. For this reason and thanks to the significant scientific and technological developments occurred in the last few decades, new renewables, e.g. solar, bioenergy, geothermal and wind, are emerging and are being the target of a great deal of researches. As a consequence, thermal and power generation industries sustained with these alternative sources of energy are becoming very important lately.

On the other hand, heat exchangers play an essential role in a wide range of applications in this kind of industries. From geothermal plants to OTEC systems or solar heaters, heat exchangers perform key duties in electricity production or domestic heating by evaporating or condensing working fluids. Due to the importance of these devices, an accurate control of the output temperatures is essential to work at full capacity and to meet the industrial requirements.

Heat exchangers can be considered as multivariable systems because the aim is to control more than one output temperature by manipulating several variables. Due to this multivariable condition and the presence of external disturbances and model uncertainties, a robust methodology based on Quantitative Feedback Theory is proposed in here to improve reliability and control performance in terms of disturbance rejection.

The paper addresses the problem of external disturbance rejection at plant output in a solar water heating system described by a 2x2 transfer matrix [1]. The desired specifications of the closed loop system (disturbance rejection and robust stability) must be achieved despite the severe coupling and the large parametric uncertainty of the process.

Different solutions were proposed in the literature to deal with different variations of this problem [2], [3], [4], [5], [6], [7]. The approach applied in this paper [8] integrates previous techniques [6], [9], [10] and designs a fully populated controller for multivariable systems.

The remainder of this paper is organized as follows. Next section presents the mathematical model of the heat application, followed by the required design specifications. Third section intends to review briefly those principles of the QFT methodology that are considered particularly useful. The following section goes into detail on the procedure to design a non-diagonal controller for external disturbance rejection. This section also includes a complete description of the system transmission matrix **T**, which relates the outputs (y) to output external disturbances (d_o). Furthermore guidelines for the design of the controller are provided. Section 5 is the point at which a controller for the heat solar process system is designed. Afterwards, in section 6, the simulation results show the performance of the designed fully populated controller comparing to a diagonal one. Finally, the most relevant ideas of the paper are summarized.

2. Model of a heat exchanger of a solar water heating system

This section presents a solar thermal energy application. It treats the use of solar heating for domestic hot water supplies. Description of the system, components and design consideration are outlined.

The basic elements of solar water heaters can be presented in several configurations systems. There is very often a heat exchanger between the collector and a storage tank, as shown Fig. 1, when antifreeze solutions are used in collectors of solar industrial processes.

The mathematical models for the key components in solar energy systems, i.e., collectors, storage units and heat exchangers are developed [1]. The manipulated variables to the whole model are q_c and q_t (collector and storage pump volumetric flux rates). The controlled variables are T_t, T_0, temperatures of the storage tank and at the exit of the collector respectively.

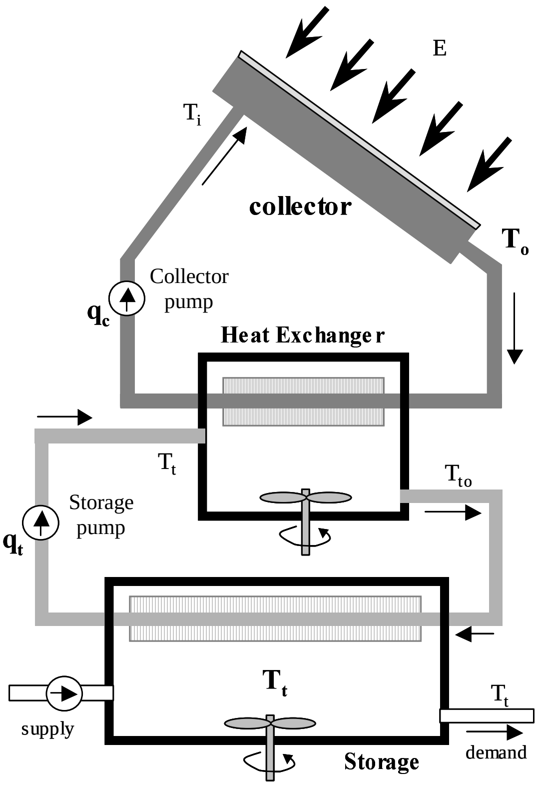


Figure 1 – Schematic of a solar water heating system

A detailed analysis of a solar process system is a complicated problem. Nevertheless a simplified analysis yields very useful results when focusing on control purposes. These results show the important variables, how they are related and how they affect to the control of the process.

The collector shown in Fig.1 heats an antifreeze solution (glycol). It is connected to a water storage tank through a heat exchanger.

The system is modelled by the following expressions,

$$Q_u(t) = \dot{m}_c\, c_{pc}\, (T_o(t) - T_i(t)) = = A_c\, F_r\, [E(t) - U_L\, (T_i(t) - T_a(t))] \tag{1}$$

$$Q_{ex}(t) = \varepsilon\, \dot{m}_t\, c_{pt}\, (T_o(t) - T_t(t)) = L_t(t) \tag{2}$$

$$T_o(t) - T_i(t) = \varepsilon\, (T_o(t) - T_t(t)) \tag{3}$$

$$\dot{m}_t\, c_{pt}\, \frac{dT(t)}{dt} = = \dot{m}_c c_{pc} (T_o(t) - T_i(t)) - U_t (T_t(t) - T_a(t)) - L_t(t) \tag{4}$$

(See the nomenclature appendix at the end of the paper)

The above set of equations describes the performance of the solar process. Eq.(1) models the collector. Eqs.(2) and (3) are related to the effectiveness of the heat exchanger. The storage unit is defined by Eq.(4).

A linearized model is then obtained around the operating point, $Q_t^o, Q_c^o, T_t^o, T_c^o$. The final expressions are combined and translated into the s-domain using Laplace transform to calculate the controlled variables (T_t and T_o) in terms of the manipulated ones (Q_c and Q_t).

$$\begin{bmatrix} \Delta T_t(s) \\ \Delta T_o(s) \end{bmatrix} = \begin{bmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{bmatrix} \begin{bmatrix} \Delta Q_t(s) \\ \Delta Q_c(s) \end{bmatrix} \tag{5}$$

where,

$$p_{11}(s) = \frac{\Delta T_t(s)}{\Delta Q_t(s)} = \frac{k_{11}}{s+\tau} \tag{6}$$

$$p_{12}(s) = \frac{\Delta T_t(s)}{\Delta Q_c(s)} = \frac{k_{12}}{s+\tau} \tag{7}$$

$$p_{21}(s) = \frac{\Delta T_o(s)}{\Delta Q_t(s)} = \frac{k_{21}}{s+\tau} \tag{8}$$

$$p_{22}(s) = \frac{\Delta T_o(s)}{\Delta Q_c(s)} = \frac{k_{22}\, (s+\gamma_{22})}{s+\tau} \tag{9}$$

and,

$$\tau = \frac{MA + ZC}{Aa}$$

$$k_{11} = \frac{Y}{A} \qquad k_{12} = \frac{-ZB}{Aa}$$

$$k_{21} = \frac{-YC}{Aa} \qquad k_{22} = \frac{-B}{A}$$

with,

$$A = b\varepsilon Q_c^o + f\, U_c\, (1-\varepsilon)$$

$$B = b\varepsilon\, (T_o^o - T_t^o)$$

$$C = f\, U_c\, \varepsilon - b\varepsilon Q_c^o$$

$$Y = \varepsilon\, d\,(T_t^o - T_o^o)$$
$$Z = -f\, U_c\,(1-\varepsilon) - \varepsilon\, d\, Q_c^o$$
$$M = U_t + f\, U_c\, \varepsilon - \varepsilon\, d\, Q_t^o$$
$$a = \rho_t c_{pt} V_t$$
$$b = \rho_c c_{pc}$$
$$d = \rho_t c_{pt}$$
$$f = A_c F_r$$

Equilibrium point:

$Q_t^o = 0.864\ 10^{-3}\ m^3\ s^{-1}$; $Q_c^o = 0.00115\ m^3\ s^{-1}$;

$T_t^o = 35\ ºC$; $T_o^o = 53ºC$;

It must be noted that each transfer function of **P** is described by a set of plants that present the natural parametric uncertainty indicated in Table I.

TABLE I. - Coefficients of parametric uncertainties

Parameter	Min	Max
ε	0.4	0.6
U_c [$w\ ºC^{-1}\ m^{-2}$]	6	8
A_c [m^2]	4	10

A disturbance could be a change in feed temperature or rate, a change in pressure or a variation in product demand. They are variables that fluctuate and cause the process output (temperatures T_t and T_o) to move from the desired operating value. The aim of the paper is to enhance the disturbance rejection performance in the MIMO system that describes the solar process so that disturbances will be a critical issue in the design.

Performance specifications:

Once the process model is developed, the desired closed-loop performance specifications are determined:

- Robust stability in each channel:

$$\left|\frac{L_i(s)}{1+L_i(s)}\right| \le 1.2 \quad i = 1, 2 \tag{10}$$

where $L_i(s)=p_{ii}(s)g_{ii}(s)$

This means at least 50º lower phase margin and at least 1.833 (5.26 dB) lower gain margin.

- Reduction of coupling effect as much as possible.
- Robust disturbance rejection at plant output so that,

$$\frac{y_i(s)}{d_{oi}(s)} \le 0.1, \ \ \omega < 0.05\ rad/s, \ \ i = 1,2 \tag{11}$$

A MIMO methodology based on QFT is proposed to cope with all the considerations described above and to obtain a robust controller.

3. QFT Theory

QFT (Quantitative Feedback Theory), has been and is one of the most successful robust control theories applied to the real world problems [9]. It is an engineering method that uses frequency domain concepts to satisfy performance specifications and handle plant uncertainty. This method relies on the observation that the feedback is needed principally when the plant presents uncertainty or when there are uncertain inputs acting on the plant.

QFT, first introduced by Isaac Horowitz [11] in 1959, is a control system design technique, a frequency domain method that uses the Nichols chart in order to achieve a desired robust design for plants having structured parameter uncertainties.

Its main objective is to design a simple, low-order controller with minimum bandwidth that satisfies performance specifications despite the variations in the model or the presence of disturbances. (For more information see [12], [13], [6], [14], [15])

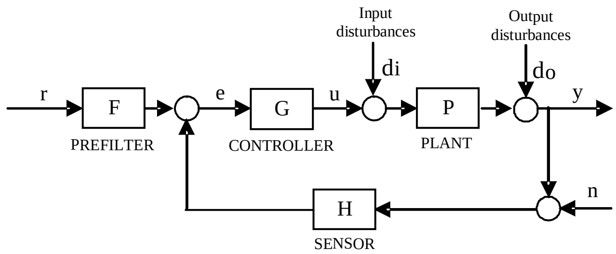


Figure 2.- 2 DOF Feedback system

4. Rejection of external disturbances at plant output in uncertain MIMO systems

The Quantitative Feedback Theory (QFT) is now applied to design a fully populated matrix controller to attenuate the effect of the external disturbances that affect the solar system presented in the previous section.

A sequential design methodology for non-diagonal QFT controllers [6], [8], [10], is applied to reject the disturbance specifications established, taking into account the reduction of interactions among loops.

Consider a *n x n* linear multivariable system like the one shown in Fig.2. The external disturbances at plant output is represented by d_o. The closed loop transfer function matrix from external disturbances at plant output d_o to the output y is called $\mathbf{T}_{Y/do}$ and it is obtained from,

$$y = (\mathbf{I} + \mathbf{P}\ \mathbf{G})^{-1} d_o = \mathbf{T}_{Y/do}\ d_o \tag{12}$$

Hence,

$$\mathbf{T}_{Y/do} = (\mathbf{I} + \mathbf{P}\ \mathbf{G})^{-1} \tag{13}$$

The expression of $\mathbf{T}_{Y/do}$, - Eq. (13) - is the starting point of a mathematical development that lead to solve this disturbance rejection problem [8].

The plant inverse $\mathbf{P}^{-1}$ will be denoted as $\mathbf{P}^* = [\, p^*_{ij} \,]$ and it is partitioned to the form $\mathbf{P}^* = \mathbf{\Lambda} + \mathbf{B}$, where Λ and $\mathbf{B}$ are the diagonal part and the balance of $\mathbf{P}^{-1}$, respectively. In the same way, the fully populated controller $\mathbf{G} = [g_{ij}]$ is divided into two terms; $\mathbf{G}_d$ and $\mathbf{G}_b$, which represent the diagonal part -subscript d- and balance -subscript b- of $\mathbf{G}$.

Substituting these matrixes in Eq. (13), operating and rearranging it, yields the next expression which describes the *n x n* matrix

$$\mathbf{T}_{Y/do} = (\mathbf{I} + \Lambda^{-1}\mathbf{G})^{-1} + (\mathbf{I} + \Lambda^{-1}\mathbf{G}_d)^{-1}\Lambda^{-1}[\mathbf{B} - (\mathbf{B} + \mathbf{G}_b\ \mathbf{T}_{Y/do})] \qquad (14)$$

By inspecting Eq. (14) a diagonal term and a non-diagonal term can be found.

i. Diagonal term $\mathbf{T}_{Y/do\text{-}d}$

$$\mathbf{T}_{Y/do\text{-}d} = (\mathbf{I} + \Lambda^{-1}\mathbf{G}_d)^{-1} \qquad (15)$$

where

$$\mathbf{T}_{Y/do\text{-}d} = [t_{do\text{-}ii}] = \left[\frac{p^*_{ii}}{g_{ii} + p^*_{ii}}\right] \qquad (16)$$

As illustrated in Fig. 3, this diagonal term is equivalent to a set of *n* MISO systems.

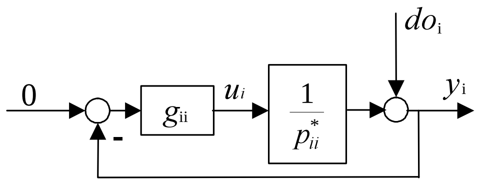


Fig. 3.- i-th equivalent MISO system. $1 \le i \le n$

ii. Non-diagonal term $T_{Y/do\text{-}b}$

$$\mathbf{T}_{Y/do\text{-}b} = (\mathbf{I} + \Lambda^{-1}\mathbf{G}_d)^{-1}\Lambda^{-1}\ [\mathbf{B} - (\mathbf{B} + \mathbf{G}_b)]\,\mathbf{T}_{Y/do} \qquad (17)$$

where,

$$\mathbf{T}_{Y/do\text{-}b} = [t_{do\text{-}ij}] = \frac{c_{do-ij}}{g_{ii} + p^*_{ii}} \qquad (18)$$

The last term in square brackets in Eq. (17) is the only part which has a non-diagonal structure. Since it depends on the balance of the controller and plant, it comprises the coupling and represents the interaction between loops. Consequently this term will be called the coupling matrix for the rejection of external disturbances at plant output and will be denoted as $\mathbf{C}_{do}$.

$$\mathbf{C}_{do} = [\, c_{do-ij} \,] = \mathbf{B} - (\mathbf{B} + \mathbf{G}_b\ \mathbf{T}_{Y/do}) \qquad (19)$$

Fig. 4 presents the block diagram of the ith control loop.

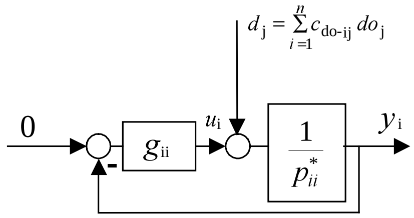


Fig. 4.- Equivalent MISO regulator with disturbances at the plant input and output. $1 \le i \le n$

Each element of the coupling matrix obeys,

$$c_{do-ij} = p^*_{ij}(1-\delta_{ij}) - \sum_{k=1}^{m}(p^*_{ik} + g_{ik})\ t_{kj}\,(1-\delta_{ik}) \qquad (20)$$

where δ_{ki} is the delta of Kronecker defined as,

$$\delta_{ki} = \begin{cases}\delta_{ki} = 1 \Leftrightarrow k = i\\ \delta_{ki} = 0 \Leftrightarrow k \ne i\end{cases} \qquad (21)$$

Now, one hypothesis and two simplifications are stated in order to make the quantification of coupling effects easier.

<u>Hypothesis H1</u>: The diagonal elements t_{jj} in Eq. (15) are assumed to be much larger than the non-diagonal ones t_{kj},

$$\left|\ t_{jj}\left(p^*_{ij} + g_{ij}\right)\right| >> \left|\ t_{kj}\left(p^*_{ik} + g_{ik}\right)\right| \quad \text{for } k \ne j \qquad (22)$$

<u>Simplification S1</u>: Applying Hypothesis H1, Eq. (20) can be rewritten as,

$$c_{di-ij} = t_{jj}(p^*_{ij} + g_{ij}) \quad ; \ i \ne j \qquad (23)$$

<u>Simplification S2</u>: The elements t_{jj} can be replaced using the expression obtained from the equivalent system, $[t_{do\text{-}ij}]$ in Eq. (16).

Applying them, the final expression of the coupling effect can be written as,

$$c_{do-ij} = p^*_{ij} - \frac{p^*_{jj}\ (p^*_{ij} + g_{ij})}{(p^*_{jj} + g_{jj})} \quad ; \ i \ne j \qquad (24)$$

Note, that every uncertain plant p^*_{ij} is represented by the following family,

$$\left\{p^*_{ij}\right\} = p^{*N}_{ij}\left(1 + \Delta_{ij}\right) \qquad 0 \le \Delta_{ij} \le \Delta p^*_{ij} \quad \text{for } i,j=1,\dots,n \qquad (25)$$

Where p^{*N}_{ij} is the nominal plant and Δ_{ij} the non parametric uncertainty radii.
In order to find out the optimum non-diagonal controller, Eq. (24) is made equal to zero and a nominal plant that minimises the maximum non-parametric uncertainty radii Δp^*_{ij} in Eq. (25) is chosen,

$$g_{ij}^{opt} = g_{jj} \frac{p_{ij}^{*N}}{p_{jj}^{*N}} \quad (26)$$

Finally, the minimum and the maximum achievable coupling effects are computed using an analogous procedure to that presented in the previous section.

$$\left| c_{ij} \right|_{g_{ij}=g_{ij}^{OPT}} = \left| \frac{p_{ij}^{*N}}{\left(1+\Delta_{jj}\right) p_{jj}^{*N} + g_{jj}} \left(\Delta_{ij} - \Delta_{jj}\right) \right| \quad (27)$$

In the same manner, the maximum coupling effect without any non-diagonal elements in the controller expression is,

$$\left| c_{ij} \right|_{g_{ij}=0} = \left| \frac{p_{ij}^{*N}}{\left(1+\Delta_{jj}\right) p_{jj}^{*N} + g_{jj}} \left(1+\Delta_{ij}\right) \right| \quad (28)$$

The design method is a sequential procedure closing loops [7], that uses fully populated matrix controllers. The end of this section outlines the step that must be followed in order to complete the whole procedure.

In order to use the design equations developed in the preceding section, firstly it is necessary to fulfil the Hypothesis H1. And secondly, another Hypothesis H2 is stated.

Hypothesis H2: The plant **P** and its inverse $\mathbf{P}^*$ should be stable and do not have any hidden unstable mode. This is only a sufficient condition to guarantee the stability of the system. Consequently, the designer must pay close attention to systems with non minimum phase or unstable elements [7], [16].

Related to the stability problem and taking into account the analyses found in several works [17], [6], [18] it can be settled that it is necessary and sufficient that the plant of each successive loop is stabilised.

In addition, before starting the sequential procedure, it is advisable to analyse the effect of interactions in the system and identify input-output pairings using the Relative Gain Array (RGA), [19]. Afterwards, matrix $\mathbf{P}^*$ is rearranged so that $(p_{11}^*)^{-1}$ has the smallest phase margin frequency, $(p_{22}^*)^{-1}$ the next smallest phase margin frequency, and so on, [6].

Then, the design technique, composed of n stages, as many as loops, performs the following steps for every column of the matrix controller **G**.

Step A: Design the diagonal element of the controller g_{kk} for the inverse of equivalent plant described in Eq.(29), using a standard QFT loop-shaping method, [13], [6].

$$\left[p_{ii}^{*e}\right]_k = \left[p_{ii}^*\right]_{k-1} - \frac{\left(\left[p_{i(i-1)}^*\right]_{k-1} + \left[g_{i(i-1)}\right]_{k-1}\right)\left(\left[p_{(i-1)i}^*\right]_{k-1} + \left[g_{(i-1)i}\right]_{k-1}\right)}{\left[p_{(i-1)(i-1)}^*\right]_{k-1} + \left[g_{(i-1)(i-1)}\right]_{k-1}}$$

$$i \geq k; \quad \left[\mathbf{P}^*\right]_{k=1} = \mathbf{P}^* \quad (29)$$

Eq.(29) represents the equivalent open-loop transfer function of the channel i^{th} assuming the previous ones have been closed. Note that the expression depends on both diagonal and non-diagonal elements of the controller.

Step B: Design the (n-1) non-diagonal elements g_{ik} ($i \neq k$, i = 1,2,...n) of the k^{th} controller column, minimising the coupling $c_{do\text{-}ik}$ described in Eq. (24) and applying the optimum non-diagonal controller equation Eq. (26).

5. Controller design for a heat exchanger of a solar water heating system

The previous methodology will be applied to the solar water heating system described in section 2 to design a robust controller which copes with the external disturbances at plant output stated formerly.

First of all the RGA (Relative Gain Analysis) [19] is calculated. Computing it for more than 600 plants generated due to uncertainty, the results show that the best possible pairing are: $\left[T_t^o - Q_t^o\right]\left[T_c^o - Q_c^o\right]$.

Design Procedure:

Step A.1: Standard QFT loop-shaping for $\frac{1}{p_{11}^*}$

$$g_{11}(s) = \frac{-0.0002s - 1.2\cdot 10^{-5}}{s^2 + 0.6s} \quad (30)$$

Step B-1: By substituting in Eq. (27) the optimum non-diagonal controller results,

$$g_{21}(s) = \frac{0.001}{s+5} \quad (31)$$

Step A-2: Once the first column (g_{11} and g_{21}) has been designed, the equivalent plant of the second channel Eq. (32) is calculated.

$$\left[p_{22}^{*e}\right]_2 = \left[p_{22}^*\right]_1 - \frac{\left(\left[p_{21}^*\right]_1 + \left[g_{21}\right]_1\right)\left(\left[p_{12}^*\right]_1 + \left[g_{12}\right]_1\right)}{\left[p_{11}^*\right]_1 + \left[g_{11}\right]_1} \quad (32)$$

Now, the diagonal controller g_{22} for $\frac{1}{p_{22}^{*e}}$ using a standard QFT loop- shaping method is,

$$g_{22}(s) = \frac{-0.06s - 0.006}{s^2 + 7s} \quad (33)$$

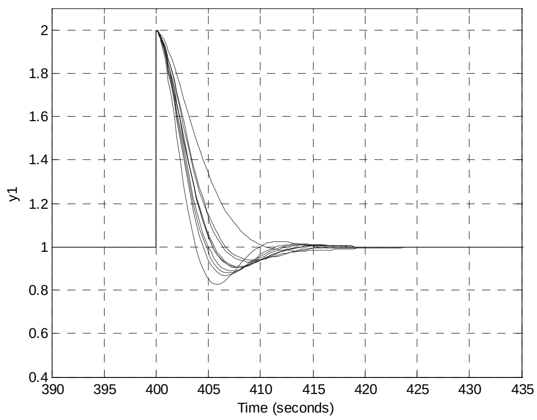
Step B-2: Design g_{12} from Eq. (27)

$$g_{12}(s) = \frac{-0.001(s+0.005)}{s+0.2} \tag{34}$$

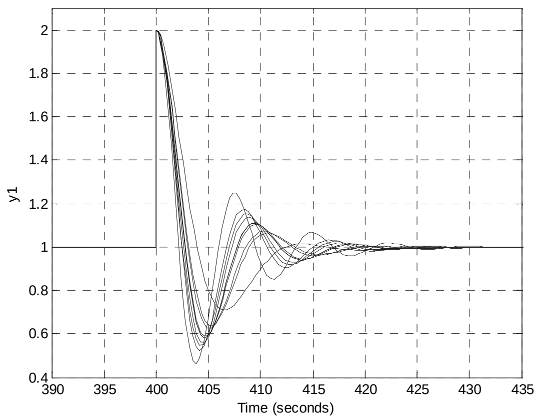
6. Results

Simulations are necessary in order to have confidence in the calculations developed in the paper. They provide a wealth of information about the plant behaviour.

The transient responses of the closed-loop system to external disturbances at plant output in the first loop are shown in Figs. 5 and 6. In case (a), a fully populated matrix controller designed with the described methodology is implemented, whereas in case (b) an only diagonal classical controller is applied. At t = 400 sec., a unit step disturbance d_1 is added at plant output y_1. As can be seen in Figs. 5 and 6, the closed-loop response to the disturbance is much more satisfactory in the case (a) that is to say, the non-diagonal controller.

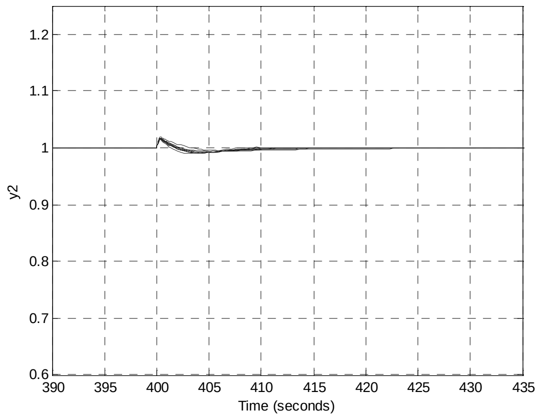


(a) Non diagonal MIMO QFT controller

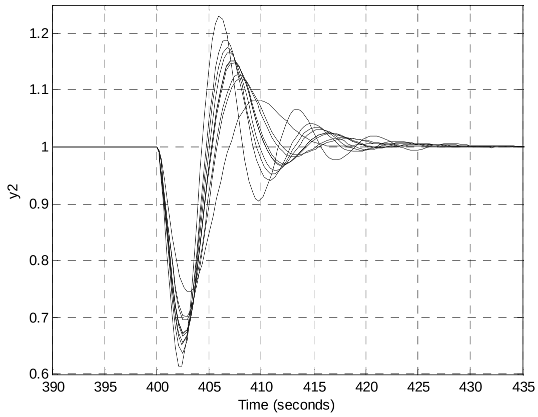


(b) Classical diagonal controller

Figure 5.- Response y_1 of the 2x2 MIMO system with a disturbance at plant output in the same channel.



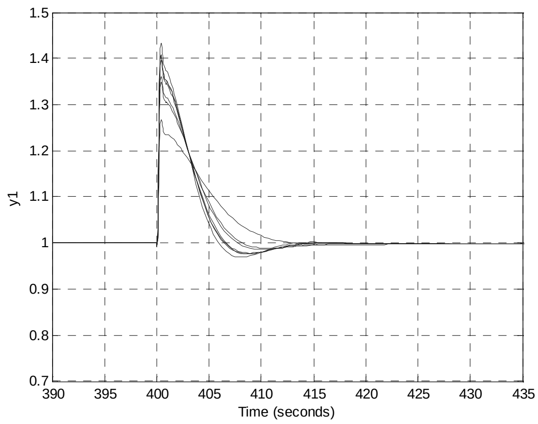
(a) Non diagonal MIMO QFT controller



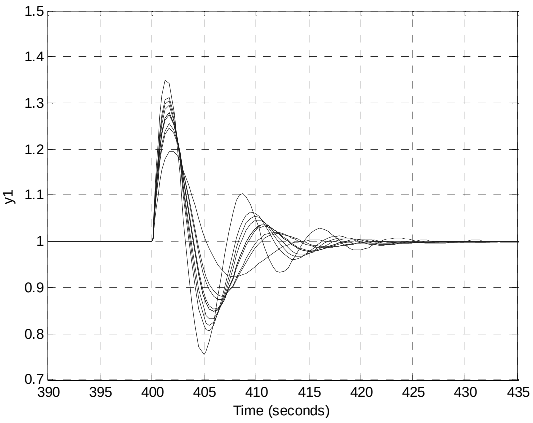
(b) Classical diagonal controller

Figure 6.- Response y_2 of the 2x2 MIMO system with a disturbance at plant output in first channel.

Figures 7 and 8 show the transient responses of the closed-loop system to external disturbances at plant output in the second loop with a fully populated matrix controller (a) and with an only diagonal controller (b) respectively. At = 400 sec., a unit step disturbance d_2 is added at plant output y_2. The results yield that the closed-loop response to the disturbance input is better, once again, with a fully populated controller.

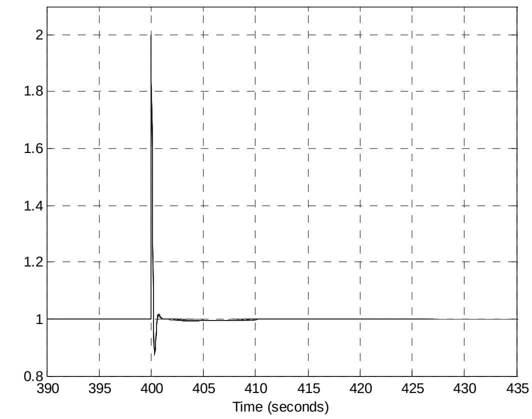


(a) Non diagonal MIMO QFT controller

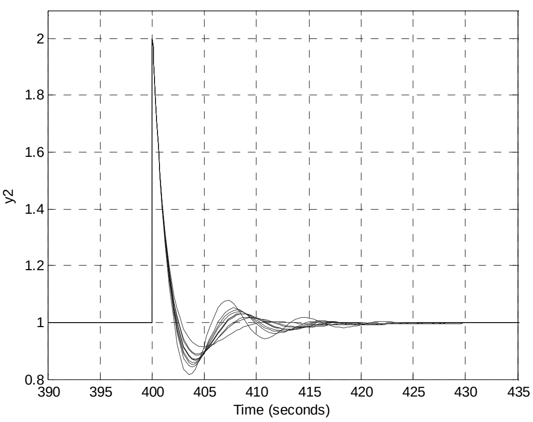


(b) Classical diagonal controller

Figure 7.- Response of the first channel (y_1) of the 2x2 MIMO system with a disturbance at plant output in the second channel.



(a) Non diagonal MIMO QFT controller



(b) Classical diagonal controller

Figure 8.-Response of the second channel (y_2) of the 2x2 MIMO system with a disturbance input in the same channel.

The diagonal controllers used to compared with are two classical structures (PI+filter) so that,

$$g_{11}(s) = \frac{-0.000533s - 8 \cdot 10^{-5}}{s^2 + 8s} \quad (35)$$

$$g_{22}(s) = \frac{-0.0057s + 0.00057}{s^2 + 10s} \quad (36)$$

7. Conclusions

This paper discussed the control of a heat exchanger placed in a solar water heating system and influenced by external disturbances at plant output. Due to the multivariable condition of the heat exchangers (several temperatures must be controlled with several manipulated variables) the control strategy selected was a robust methodology based on QFT. The complete design procedure was described and applied.

The approach was proven to work well for the solar application showed. The controller was found effective in achieving given specifications. It not only copes with plant uncertainties but also enhances the rejection of external disturbances. Moreover, the controller attenuates successfully the coupling between the control loops.

Some significant simulation results were presented in the paper. For the solar system, the MIMO methodology gives better control than classical diagonal controllers.

The preceding results provide preliminary indications of the feasibility of the proposed theory to reject disturbances at plant input not only in solar processes. Due to the generality of the design method, it is applicable not only to solar processes but also to many industrial heat exchangers with only changing the manipulated and control variables.

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Nomenclature

$\dot{m}_c = \rho_c \, q_c =$ Glycol flow rate [Kg s^{-1}]

$\dot{m}_t = \rho_t \, q_t =$ Water flow rate [Kg s^{-1}]

$\rho_c = 1094$ Kg m^{-3} = Glycol density

$\rho_t = 1000$ Kg m^{-3} = Water density

$q_c = 0.00115$ m^{3}s^{-1}

$q_t = 0.864 \; 10^{-3}$ m^{3}s^{-1}

$c_{pc} = 3850$ J Kg^{-1} ºC^{-1} = glycol specific heat

$c_{pt} = 4190$ J Kg^{-1} ºC^{-1} = water specific heat

$V_t = 0.1$ m^3 = Volume of water in the tank

$\varepsilon = [0.4\text{-}0.6]$ = heat exchanger effectiveness

$U_c = [6\text{-}8]$ w m^{-2} ºC^{-1} = overall loss coefficient

$U_t = 2500$ w ºC^{-1} = overall heat transfer coefficient area

$E =$ solar radiation

$F_R = 0.8$

$A_c = [4\text{-}10]$ m^2 = collector area