

## Piezoelectric Power Generator: a comparison of power generation between the resonant frequency and natural frequencies

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**Abstract:** The capture of energy from the environment, mostly vibrations using piezoelectric materials is of great interest, justified by the intensive use of low-power portable devices and wireless sensors, without the need to use an external power supply. This presentation is an analysis of the vibration energy in an embedded metal structure, with mass at the free end. The metallic structure is coupled to a piezoelectric plate-type PZT-4. The dimensions of the piezoelectric plate are 10x10x0.5 mm. The material used is a brass metal structure and its dimensions are 70x10x0.3 mm. A cam-follower is used to excite the electrical generator with mechanical vibration in the range of 0-30 Hz. Results indicate that the system has a capability of low power generating a range of 200 to 300 mW. A circuit conditioner treats and stores the generated signal. Still, operational analysis on resonance frequency of the structure and natural frequencies of the system will be compared, presented and discussed.

### Key words

Low power generator, piezoelectric plate, resonant frequency, natural frequencies.

### 1. Introduction

Piezoelectric materials have been widely used in environmental movement conversion mechanisms, primarily of vibrations into electrical energy. This energy may be stored or used directly to power low power devices, such as: cell phones, portable electronic devices, and wireless sensors, according to Sodano [1]. When a cell is coupled to a piezoelectric metallic structure, it creates an electromechanical system capable of capturing mechanical vibrations and transforming them into electrical energy. This type of system allows the

generation of low power electricity with applications in portable electronic devices. A correct mathematical modeling is required, because the entire dynamic of behavior reflect various parties engaged in the project. This electromechanical system typically consists of a cantilever with mass at the free end and a coupling between the metal beam and piezoelectric plate, in addition to the connection between the electrical charge to the mechanical device. These systems have functional bands and electricity catchment models presented by Sodano [2] and Rocha [3] that study the power dissipated in a resistor or a minute combination of linear electric elements. Most of these models that stress the simplification of conditioning circuits and storage of the electrical signal by direct application of a resistor may not accurately describe the real applications.

For Erturk [4], simplistic considerations may lead to incorrect physical assumptions and errors in modeling of vibration pick-up systems. The authors show best dynamic models, in addition to presenting an overview of numerical and analytical modeling of electromechanical systems. Guyomar [5], presents progress on electromechanical conversion and processing of the signal generated using non-linear electronic interfaces. One of the electric circuit models presented by Guyomar will be discussed in the following sections, is the *Hybrid Synchronized Switch Harvesting on Inductor (Hybrid SSHI)*. Wu *et. al.* [6] present an architecture similar to SSHI series, but with a change in switching control called *Synchronized Switching and Discharging to a storage Capacitor through an Inductor (SSDCI)*.

Regarding the amount of power generated, Sodano *et. al.* [7] studies were conducted to demonstrate that the output power of piezoelectric material was able to recharge a fully discharged battery without the use of external energy sources. It was verified that his system could generate a

maximum power of 2 mW when excited with the resonant frequency. It was also investigated whether other natural frequencies could charge the same battery, compared to the load time in resonant frequencies.

The feature of power generation in natural frequencies, i.e. outside the resonant frequency, arouses interest and prompted the author to research this subject. Because little is known about the physical effects of the ceramic material or on the coupling between PZT plate and metal plate, varying configurations, and the effects caused by resonant frequencies. In these cases the mechanical movements are amplified and the possibility of the present system failures is significant. Motter [8] uses the same standard circuits for the direct transfer of energy, developed by Guyomar [5]. Motter [8], in particular, presents results of electric power generation by comparing the power in resistive loads in 437 Hz and 58 Hz, resonant frequency and a natural mode of vibration, respectively. Minazara [9] applied a piezoelectric generator similar to Sodano [7] in the handlebars of a bicycle. Under real conditions, such as pure sinusoidal vibrations at  $5 \text{ m/s}^2$  acceleration and 12.8 Hz frequency, the measured values reached 3.5 mW to a resistive load of 100 K $\Omega$ . This energy is enough to recharge a battery, or for use in low power devices.

The vast majority of piezoelectric generation devices studied in the literature consist of a cantilever-free with concentrated mass on the free end, whose properties (length, mass, material) are adjusted so that their natural frequency coincides with the frequency of operation of the metal structure. The performance of these resonant devices for energy use is highly dependent on an appropriate adjustment of its frequency of resonance with the natural frequency of system operation. Therefore, any maladjustment due to variability of resonant device properties or the frequency of operation of the system can lead to great losses in their performance in terms of energy generation Godoy [10] and Adhikari [11]. The work will show values of voltage, current and power by comparing the results for resonant frequency and natural frequencies of the system.

Non-linear conversion Interfaces will be addressed from an experimental setting of the electromechanical system conversion. The hybrid electric circuit SSHI signal conditioning and energy storage of Guyomar [5] will be reproduced. Experimentally, the electrical power is measured and shown to a range of values of resistive electrical charges.

The aim of the paper is to perform an analysis of the generation system by comparing the maximum capacity power generated to its resonance frequencies and natural frequencies.

## 2. The Prototype

The design of the generation system is composed of a metallic structure embedded with mass at the free end.

The metallic structure is coupled to a piezoelectric plate-type PZT-4. The material used in the beam is Brass: ASTM - B36 - 26800 (base = Cu / Zn = 35% / Pb = 3%). The dimensions of ceramic PZT plate and brass bar are presented in Table 1. A cam follower is required to excite the piezoelectric generator with mechanical vibrations in the range of 0-30 Hz. The frequency inverter is responsible for the excitation of a motor 1/3 HP. The motor shaft is coupled to the eccentric cam follower. A piezoelectric accelerometer is used to measure acceleration. The software Matlab generates the waveform of the frequency response. An electrical circuit for conditioning and storage is presented, as well as nominal values of the electronic components.

Table 1 – Dimensions of the electromechanical transducer

Description	Dimensions (mm)	Area (mm <sup>2</sup> )
Brass beam	70 x 10 x 0,5	700
Ceramic PZT plate	10 x 10 x 0,5	100

The following show the general model, mechanical and electrical system of a piezoelectric generator.

### A. General Model of Energy Harvesting

The Fig. 1 shows the schematic diagram of the experimental setup.

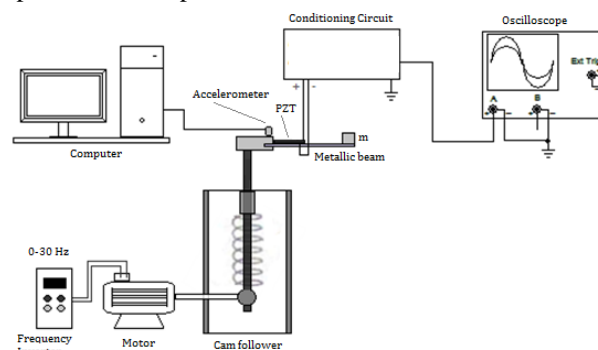


Fig. 1 – Schematic diagram of the experimental setup.

Second Guyomar [5], generally speaking, the capture of vibration energy can be represented using the diagram shown in Fig. 2. First mechanical energy (for example, mechanical power or an external acceleration) is converted into mechanical energy in the metallic structure. It is then converted into electrical energy by means of piezoelectric material and finally transferred, to an electrical energy storage device.

### B. Mechanical Modeling

The mass spring system Fig. 3 represents the mechanical equivalent model of a piezoelectric structure.

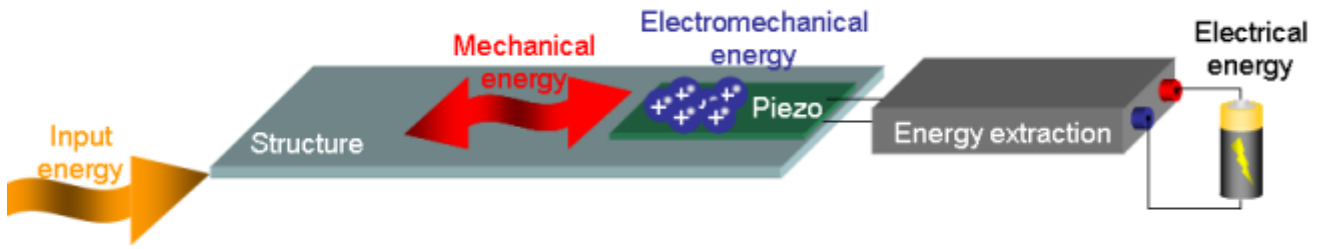


Fig. 2 - General schematic of a vibration energy harvester, by Guyomar.

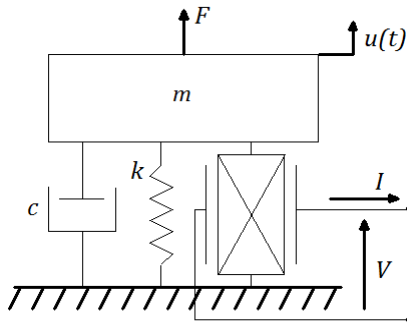


Fig. 3 – Equivalent mechanical model of a piezoelectric structure.

For the mechanical system the Fig. 4, can be considered a cantilever with a free end and mass in this same end, Rao [12].

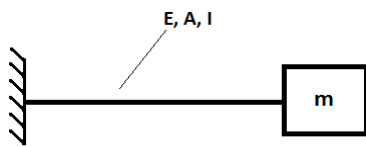


Fig. 4 – Mechanical system of a cantilever-free end.

The transverse displacement of the beam  $y(x)$  can be expressed as:

$$y(x) = \frac{Px^2}{6EI}(3\ell - x) \quad 0 \leq x \leq \ell \quad (1)$$

Where  $I$  is the moment of inertia of the cross section beam,  $P$  is the force applied in  $I$  and  $E$  is the Young's Modulus of the material.

The rigidity of the beam is described by equation (2):

$$k = \frac{P}{y(\ell)} = \frac{3EI}{\ell^3} \quad (2)$$

The natural frequency is expressed by (3):

$$\omega_n = \sqrt{\frac{k}{m}} \quad (3)$$

It is considered that the vibrating base has a harmonic motion:

$$y(t) = Y \sin(\omega t) \quad (4)$$

The equation of the movement of mass  $m$  can be written as:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \quad (5)$$

Setting the displacement relative  $z = x - y$ , the equation (5) can be rewritten:

$$m\ddot{z} + c\dot{z} + kz = -m\dot{y} \quad (6)$$

The solution in steady state is given by the equation (7):

$$z(t) = Z \sin(\omega t - \Phi) \quad (7)$$

Where  $Z$  and  $\Phi$  are given to (8) and (9):

$$Z = \frac{Y \cdot r^2}{\sqrt{(1 - r^2)^2 + (2 \cdot \xi \cdot r)^2}} \quad (8)$$

$$\Phi = \arctg\left(\frac{2 \cdot \xi \cdot r}{1 - r^2}\right) \quad (9)$$

Where

$$r = \frac{\omega}{\omega_n} \quad (10)$$

### C. Electric Modeling

The electric circuit for conditioning and storage of energy generated by PZT ceramics will be based on the circuit of Umeda *et al.* [13] *apud* Sodano *et al.* [1]. The ceramic generates electrical peaks. During their mechanical excitation, these peaks were rectified by the full bridge rectifier and filtered through a capacitor.

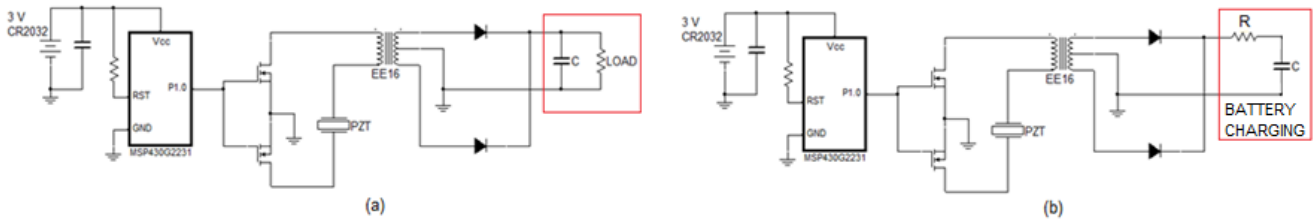


Fig. 5 - Power conditioner circuit with the microcontroller MSP430G2231. (a) with resistive load (b) with charging capacitor.

The Fig. 5 shows the complete electric circuit used for the DC signal conditioning. The MSP430G2231 microcontroller was used, because it is a low-power device. The MOSFET's performed the output signal switching in PZT ceramics, because the transformer EE16 (x10) works in high frequency. The switching frequency was 40 kHz. For the full-bridge rectifier, germanium diodes were used, due to low voltage response in this diode type. The Fig. 5(a) capacitive filter was used 330 nF and the resistive load varied in a range of values presented in Table 2.

The choice of using only two diode full wave rectification with central tap transformer was required due to low voltage across the germanium diodes. The losses in the rectifier diodes are 0,3V, in D<sub>1</sub> and D<sub>2</sub>. In Fig. 5(b) an RC circuit on the output was used to calculate the charging time of the capacitor. The different types and values of the capacitors are shown in Table 3.

### 3. Results

The basis of the study was to try to identify different frequencies of the resonance frequency that generate low power energies with similar intensities. Therefore, the data presented are based on the resonance frequency of 19.5 Hz and one of the natural frequencies of the system 2.5 Hz. The Fig. 6 shows the voltage (V) generated on experimental piezoelectric generator system and measured with an oscilloscope.

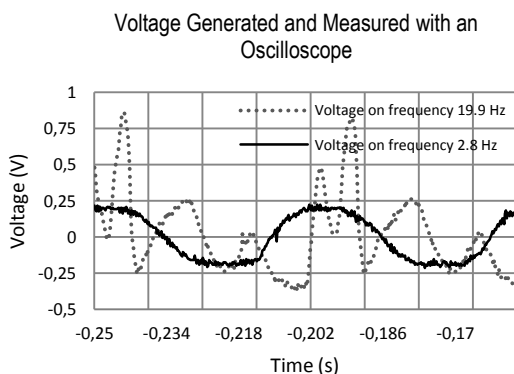


Fig. 6 - Voltage generated and measured on experimental system.

It also indicated a waveform frequency of 2.5 Hz most periodic, this signal can be handled more appropriately by the electronic components of the conditioning circuit.

The Fig. 7 presents the acceleration of the system cam follower measured with accelerometer.

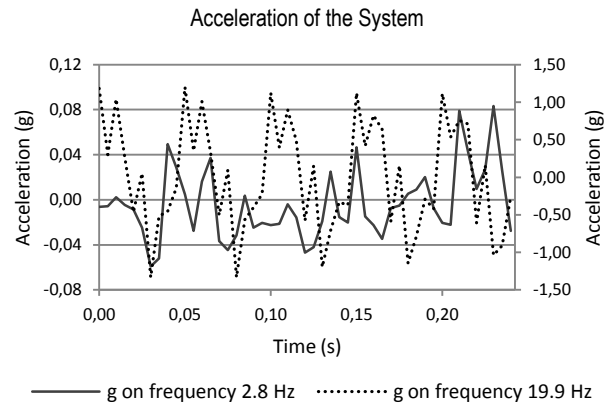


Fig. 7 - Vibration of the system measured with accelerometer.

Although the magnitude of the acceleration to gravity is evidently higher in the resonant frequency in the Fig. 7 again shows up the fact that the periodicity of the signal is good in 2.5 Hz. Energy harvesting is more easily captured on the lower signals, not requiring specific frequencies for operation of the system.

The Fig. 8 shows the frequency response to condition forced 19.5 Hz (resonant frequency) and 2.5 Hz natural (frequency natural), as the application of g-force applied to the system.

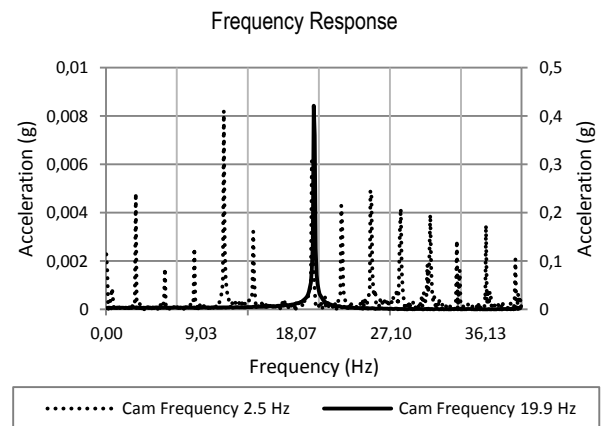


Fig. 8 - Frequency response wave.

The electromechanical system is oscillating at 2.5 Hz. The authors studied the effect of the system generator operation in the first natural frequency. The graph of the frequency response shows again that the signal amplitude at 19.5 Hz is greater than 2.5 Hz. However, this comparison is valid because the system operates in frequency ranges that are not destructive to the mechanical parts. This is one of the concerns of the study.

The Fig. 9 shows the power generated at two frequencies, at resonance 19.5 Hz and natural of 2.5 Hz.

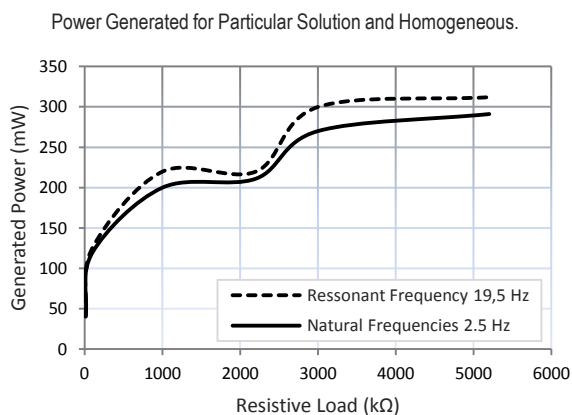


Fig. 9 - Power generated for particular solution and homogeneous at Resonant Frequency and Natural Frequency.

It can be stated that the resonant frequency of the system (19.5 Hz) sinusoidal voltage at the output of PZT has a high noise, allowing and causing malfunction of the electronic circuit requiring the inclusion of filters. In this case, the system generated 220 mW with load of 1 MΩ. For the first vibration modes of the system in its natural frequencies (2.5 Hz), the waveform of the voltage at the output of PZT is essentially a sine wave with low noise. The process of rectification, filtering and switching performed with the Circuit Hybrid SSIH generated 200 mW to a load of 1 MΩ, in the natural frequencies at 2.5 Hz.

Table 2 – Results obtained for a range of resistive loads.

Resistive Load	Generated Power at $F_R$ (19.5 Hz)	Generated Power at $F_N$ (2.5 Hz)
10 kΩ	41 mW	40 mW
100 kΩ	126 mW	120 mW
1 MΩ	220 mW	200 mW
2.2 MΩ	220 mW	211 mW
3 MΩ	300 mW	270 mW
5.2 MΩ	312 mW	291 mW

In Table 3 can observe charging time of the capacitors 1000 nF and 470 pF. In the Fig. 10 showed waveforms. Note that for the resonance frequency, the charging voltage is higher. But this does not affect charging time, that from 161.0 s reaches the full charge of the respective capacitors.

Table 3 – Results obtained for a charging time.

Capacitor	Charging Time at $F_R$ (19.5 Hz)	Charging Time at $F_N$ (2.5 Hz)
1000 nF	208,2 s	161,0 s
470 μF	183,4 s	276,2 s

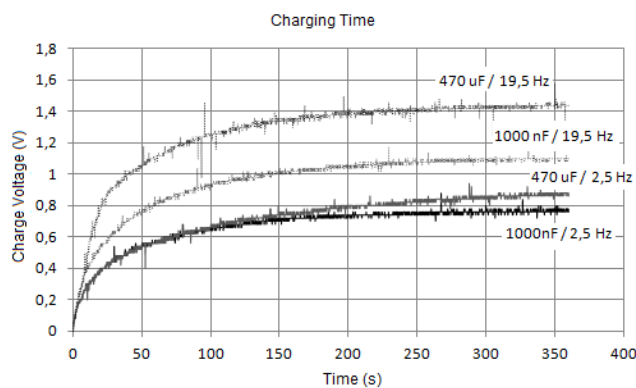


Fig. 10 – Charging time for two values of capacitors 1000 nF and 470 μF.

#### 4. Conclusion

This paper presented the behavior of a piezoelectric generator system. Was exposed to electric generation capacity outside the resonant frequency range, i.e. in other natural frequencies. Mechanically, it is important to note that the method extends the life of systems operating in non-resonant frequencies, without non-destructive mechanical parts. Proved the difference between the accelerations presented, but not compromising power output. The mean power generated in the resonant frequency at 19.5 Hz of the structure, is 220 mW. The average power generated in natural frequencies of 2.8 Hz is 200 mW for a range of resistive loads of 10 kΩ – 5.2 MΩ. The piezoelectric generator is able to generate in natural frequencies. The experiment proved to be able to charge capacitors with an average time of 200 s regardless of the frequency of excitation of the system. It is capable of charging batteries of portable electronic devices.

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