



Analytical Expression For The Tower Height of WECS

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Abstract. This paper presents an analytical expression for the optimum tower height of a wind turbine in a given site. The optimum tower height is determined such that to maximize the probability of the effective wind speed range and hence keeping the wind turbine generator working the longest possible time period in that range. A chart is also presented for graphical determination of the optimum tower height. The optimum tower height determined helps as an indicator for wind turbine-site matching; very large values, beyond the practical limits, means that the wind turbine generator characteristics do not match the wind resource at the site.

Key words

Wind energy, tower height, wind turbine-site matching

1. Introduction

The ambitious target levels of renewable energy contribution into the energy demand worldwide have impelled the renewable energy industries towards large capacity units/stations. The wind energy industry as one of the major renewable energies has grown and still growing fast towards larger turbine capacities and hence taller tower heights. Turbine sizes are averaging 2.0 – 3.0 MW for onshore projects with new turbines developed at the 4.0 – 5.0 MW sizes. For the onshore market, the 3.0 MW turbines are fast becoming the standard with 90 m tower height [1].

Unlike the conventional generation system, performance of Wind Energy Conversion System (WECS) is site dependent. The energy gathering efficiency, capacity factor, the optimum turbine speed, etc. are all site specific. Recognizing this fact, several research efforts have been reported for wind turbine-site matching [2-7]. A method for selection of the optimum Wind Turbine (WT) for a specific site based on the Capacity Factor (CF) of the available WTs is presented in [2], where the CF is calculated using long term wind speed data. A case for site matching of WTs is presented in [3], where different mean speeds are used with different probability density functions. It has been found the CF calculated from the Weibull distribution using the cubic mean of wind speed fairly match the actual CF. Risk-based capacity benefit

factors were used in [4] to determine the optimum site-matching; the effects of WT design parameters on the basic adequacy indices were also studied. Methods for identifying optimum turbine speed parameters to yield higher energy production were introduced using normalized power curves [5] and enhanced power curve strategy [6]. In [7], a new turbine-site matching index (TSMI) has been derived considering wind speed characteristics, the WT power curve parameters, and turbine size as well as tower height.

Tower is an expensive part and represents a considerable fraction of the total cost of the wind turbine. The price of a tower for a WECS is around 20% of the total cost of the WECS. Each meter of the tower costs money. For a tower around 50 m height, the additional cost of another 10 meters of tower is about US\$15000. It is therefore quite important for the final cost of energy to build towers as optimally as possible. Determination of the optimum height of the tower should consider tower costs per meter, wind variation with height, and the price the turbine owner gets for an additional kWh of electricity [2].

Cost of the WECS is not the only factor affected by tower height; the WECS performance is also greatly affected by the tower height. This is due to the fact that towers are as integral to the performance of the WECS as the wind turbine itself. It is not just a supporting structure for the turbine, generator and other components. The tower helps to place the turbine at a height where the wind is better and more energy can be gathered.

It is well known that increasing the tower height increases the wind speed. The power available in wind to be extracted will also increase as the cube of wind speed. The power output of the wind energy conversion system will also increase but it will not follow the cubic relation. This is due to the fact that as the wind speed exceeds the rated wind speed (V_r) the power output remains constant at its rated value and the excess energy available in wind is spelled away. Also, if the wind speed falls down the cut-in speed (V_{ci}) or if it increases above the cut-out speed (V_{co}) the power output from the WECS will be zero. Only in the wind speed range $V_{ci} < v < V_r$ the power output of the WECS nearly matches the increase in power available in wind. So, the energy gathering

efficiency of the WECS will have its best value if the probability that the wind speed is enclosed in the range $V_{ci} < v < V_r$ is maximum.

Keeping the WT running the maximum possible time in that range increases the energy output and the CF; hence improves the economic value of the WECS. So, the design parameters of the wind energy conversion system chosen for installation at a given site should be matched to the WSFD of the site such that the probability of $V_{ci} < v < V_r$ is maximum. However, each site has its own Wind Speed Frequency Distribution (WSFD) and even in the same region, the WSFD may differ at one location from that at another location. Adding to that the limited number of standard designs of the WECSs, it is almost impossible to find a WECS that represent a perfect match for each individual site. The WECS that represents the best match for the wind resource at one site may not be even a good match for another site.

The practicalities of manufacturing prohibit matching the WECS and the WSFD of the site through designing a special WECS for each site. The only available option is to select from the available designs the one that gives the best performance at the site. As mentioned in a previous section, different methods have been proposed for this purpose [2-7]. The turbine performance index TPI is introduced and used as a means for matching the WTG with the wind regime [8], for some sites in Egypt.

In this paper, tower height is used to match one of the already existing designs for the WECS to the WSFD of the site. This can be achieved by altering the WSFD of the site to match the existing design of the WECS. Changing the tower height at which the WECS is to be installed is the available means that can modify the WSFD.

2. Tower Height Determination

This section presents mathematical derivation of the optimum tower height for a WT at a specific site. First, the wind speed model and the related mathematical relationships are presented.

A. Wind Speed Model

The two parameters Weibull distribution is most commonly used probability distribution for describing wind speed. The Weibull probability density function at h_0 and the cumulative probability function at the same height are given by (1) and (2) respectively.

$$f_0(v) = \frac{k_0}{c_0} \left(\frac{v}{c_0}\right)^{k_0-1} \cdot e^{-\left(\frac{v}{c_0}\right)^{k_0}} \quad (1)$$

$$F_0(v) = 1 - e^{-\left(\frac{v}{c_0}\right)^{k_0}} \quad (2)$$

Where v is the wind speed in m/s, k_0 is the shape parameter and c_0 is the scale parameter of the Weibull distribution representing wind speed at height h_0 . The values of c_0 and k_0 are related to mean wind speed, m_0 , and the standard deviation of wind speed, σ_0 , by the following equations [9]

$$c_0 = \frac{m_0}{\Gamma(1+1/k_0)} \quad (3)$$

$$\left(\frac{\sigma_0}{m_0}\right)^2 = \frac{\Gamma(1+2/k_0)}{\Gamma^2(1+1/k_0)} - 1 \quad (4)$$

B. Effect of Tower Height on WSFD

To be able to determine the probability distribution function at a new height h it is important to determine the shape and scale parameters at the new height, k_h and c_h respectively, as functions of those at height h_0 , k_0 and c_0 . The simplest, yet most commonly accepted, form for the wind speed variation with height is the power law equation expressed as follows.

$$\frac{v_h}{v_0} = \left(\frac{h}{h_0}\right)^\alpha \quad (5)$$

Where v_0 is wind speed at the measurement height h_0 , and v_h is the corresponding wind speed at height h . The friction coefficient α depends on surface roughness and is determined empirically. Typical values of α for different terrain types are listed in table I. However, an average value of α is determined from several measurements for different sites around the world and found to be 1/7 [8]. Therefore, α is assumed equal to 1/7 for the illustrative calculations unless otherwise stated.

Table I
Friction Coefficient α For Different Terrain Types

Terrain type	α
Lake, ocean and smooth hard ground	0.10
Foot high grass on level ground	0.15
Tall crops, hedges, and shrubs	0.20
Wooded country with many trees	0.25
Small town with some trees and shrubs	0.30
City area with tall buildings	0.40

Equation (5) can be rewritten as follows.

$$v_h = v_0/x \quad (6)$$

where: $x = (h_0/h)^\alpha$

The effect of height above ground on the statistical parameters of wind speed can also be determined. The mean wind speed at height h , m_h , can be determined from wind measurements at h_0 by correcting each measurement to the height h using (6); it is found to be as follows.

$$m_h = \frac{1}{N} \sum_{i=1}^N V_{hi} = \frac{1}{N} \sum_{i=1}^N \frac{v_{0i}}{x} = \frac{m_0}{x} \quad (7)$$

The standard deviation of wind speed at height h , σ_h , can also be proven to be

$$\sigma_h = \frac{\sigma_0}{x} \quad (8)$$

Equations similar to (3) and (4) can be written for c_h

and k_h as follows.

$$c_h = \frac{m_h}{\Gamma(1+1/k_h)} \quad (9)$$

$$\left(\frac{\sigma_h}{m_h}\right)^2 = \frac{\Gamma(1+2/k_h)}{\Gamma^2(1+1/k_h)} - 1 \quad (10)$$

Substituting for m_h and σ_h from (7) and (8) into (9) and (10), we get

$$c_h = \frac{m_o/x}{\Gamma(1+1/k_h)} \quad (11)$$

$$\left(\frac{\sigma_o}{m_o}\right)^2 = \frac{\Gamma(1+2/k_h)}{\Gamma^2(1+1/k_h)} - 1 \quad (12)$$

Comparing (12) with (4) it can be found that $k_h=k_o$. Comparing (11) with (3), putting $k_h=k_o$, leads to $c_h=c_o/x$. Hence the Weibull distribution WSFD and CDF at height h can then be expressed as follows.

$$f_h(v) = \frac{x k_o}{c_o} \left(\frac{xv}{c_o}\right)^{k_o-1} \cdot e^{-\left(\frac{xv}{c_o}\right)^{k_o}} \quad (13)$$

$$F_h(v) = 1 - e^{-\left(\frac{xv}{c_o}\right)^{k_o}} \quad (14)$$

It is clear now that Weibull distribution functions at height h are related to those at h_o as follows.

$$f_h(v) = x f_o(xv) \quad (15)$$

$$F_h(v) = F_o(xv) \quad (16)$$

The relationships of (15) and (16) can also be proven valid for distributions other than Weibull distribution; hence the results obtained using these two equations are applicable to any of the distribution functions used for describing the wind speed frequency distribution.

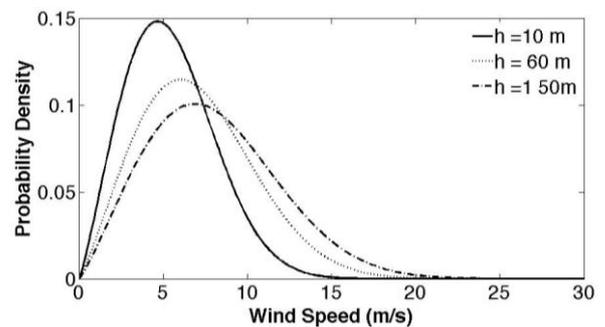
Fig. 1 shows Weibull PDF, 1.a, and CPF, 1.b, of wind speed for a site at the measurement height of 10 m. Weibull scale parameter and shape parameter of the site at this height are 6.166 m/s and 2.197 respectively. The figure also shows the PDF of wind speed at heights of 60 m and 150 m. The effect of height on the wind speed PDF is clear. However, the following values taken from these figures can make the effect of height more clear. The maximum probability densities for the 3 cases are 0.14842, 0.11490 and 0.10081 for the 10 m, 60 m and 150 m heights respectively; the corresponding wind speeds are 4.7 m/s, 6 m/s and 6.9 m/s respectively. Hence, it can be said that the PDF at higher heights gets flattened and stretched towards higher wind speeds, the probability of low wind speeds gets smaller and the probability of high wind speed gets higher.

To show how the height would affect the performance of a WT at a site, a WT having V_{ci} , V_r and V_{co} of 4 m/s, 15 m/s and 25 m/s is used. At wind speeds below V_{ci} ($v < V_{ci}$) or higher than V_{co} ($v > V_{co}$), the WT output is zero. For wind speeds between V_{ci} and V_r ($V_{ci} < v < V_r$), the WT output follows the power in the wind; whereas between V_r and V_{co} ($V_r < v < V_{co}$), the WT output is kept constant at its rated power. Table II lists the probability and duration of the wind speed being in any of these ranges for the tower

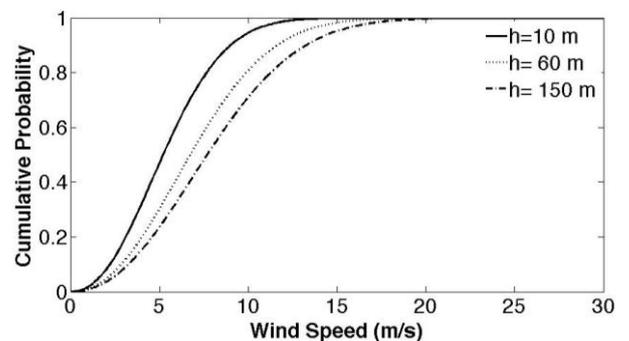
heights considered in Fig. 1. It can be noticed that increasing the height reduces the number of hours below V_{ci} and increases the number of hours in the higher speed ranges.

Table II:
Probability and Duration of The Different Wind Speed Ranges

	H=10	H=60	H=150
Prob($v < V_{ci}$)	0.3205	0.1977	0.1523
Duration (hr/year)	2808	1731	1334
Prob($V_{ci} < v < V_r$)	0.6786	0.7844	0.7986
Duration (hr/year)	5944	6871	6996
Prob($V_r < v < V_{co}$)	0.0008	0.0179	0.049
Duration (hr/year)	8	158	429
Prob($v > V_{co}$)	3.9×10^{-10}	4.36×10^{-6}	9.53×10^{-5}
Duration (hr/year)	0	0	1



(a). Weibull PDF at different heights



(b). Weibull CPF at different heights

Fig. 1. Effect of height on the Weibull distribution of wind speed

It can also be noted that the number of hours in the range ($V_{ci} < v < V_r$) represents the majority of the duration in the whole power producing range; these represents 99.87%, 97.75% and 94.21% of the total WT operation hours at the 10 m, 60 m, and 150m heights respectively. Hence, this wind speed range ($V_{ci} < v < V_r$) is the dominant range, which has greatest effect on the WT performance. The number of hours in this range increases with height; hence, it can be said that the taller the tower the better as long as the technical and economical considerations allow it. This statement may be true to some extent, however examining the effect of tower height on the duration of the WT operation range reveals some limitations on its truth.

Fig 2 shows the marginal increase in duration of the wind speed range ($V_{ci} < v < V_r$) at tower heights between 30 m and 300 m for the WT described before as well as for two different values of V_{ci} . It is clear from the figure that the marginal increase in duration decreases as the tower height increases. This can also be observed from table II, where an increase of 50 m (from 10m to 60m) in height causes an increase of 927 hours in the duration of the range ($V_{ci} < v < V_r$), while a further 90 m increase (from 60m to 150 m) causes an increase of only 125 hours. The figure also shows that the marginal increase in duration keeps decreasing until it becomes zero at a height, dependant on the WT parameters, and then turns to be negative beyond that height. This means that there is a tower height at which the probability of ($V_{ci} < v < V_r$) and hence the duration of this range reaches a maximum value. This is illustrated in Fig. 3, where the probability of ($V_{ci} < v < V_r$) is plotted against the tower height. It can be noticed that there is a tower height at which the probability is maximum. It can be noticed also that the tower at which the maximum probability occurs is not the same for different values of V_{ci} . It can be noticed that lower V_{ci} results in lower optimum height. Optimum height will be used in the rest of the text to refer to the height at which the probability of ($V_{ci} < v < V_r$) is maximum. P will be used to refer to the probability of ($V_{ci} < v < V_r$). Lower V_{ci} results also in higher value for P.

In Fig. 4, the probability P is drawn against the tower height for different values of V_r . The effect of V_r on the optimum tower height and the value of the maximum probability is also clear. The effect of V_r is more noticeable at larger tower heights. It can be noticed that higher values of V_r results in higher P at the expense of higher tower. It is, however, not in the interest of this work to set the value of V_{ci} or V_r for the WT; the interest is, however, matching an existing WT parameters to a site with known WSFD.

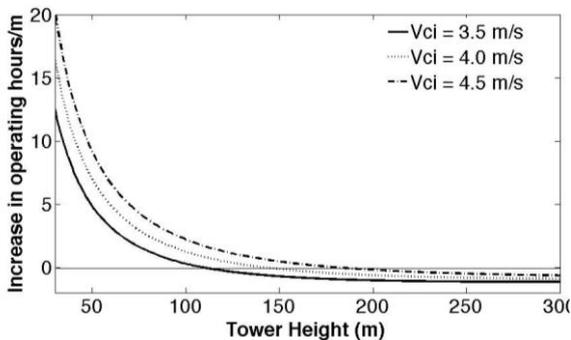


Fig. 2. Marginal duration of the wind speed range ($V_{ci} < v < V_r$).

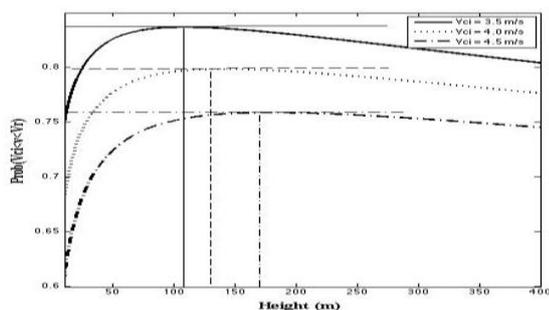


Fig. 3. Effect of tower height on the probability of ($V_{ci} < v < V_r$).

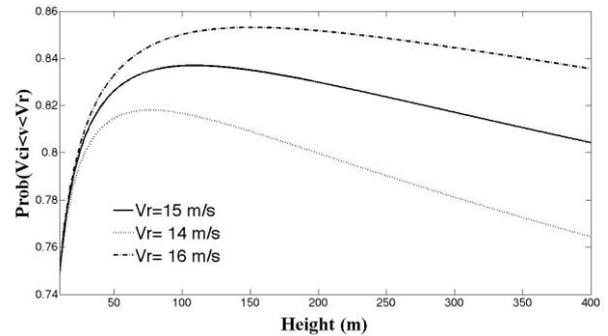


Fig. 4. Effect of V_r on the probability of ($V_{ci} < v < V_r$).

The discussion in the preceding sections may have clarified that it is not an easy task to set up simple rules of thumb about tower height selection. An optimum tower height is to be determined for each case separately. In the next section a formula for the optimum tower height is derived analytically. The optimum tower height is derived as a function of both the site WSFD parameters and the WT parameters.

C. Optimum Tower Height

It should be mentioned that the maximum duration in the effective wind speed range is highly desirable for the overall energy harvesting efficiency, however it is not the only factor that determines the optimum tower height. The criterion for determining the most suitable tower height is to maximize the probability of the effective wind speed range, $V_{ci} < v < V_r$, as in this range all the power available from wind can be fully extracted by the WECS.

Thus if a WECS is to be installed at a site whose wind speed frequency distribution and cumulative distribution functions at reference height h_0 are $f_0(v)$ and $F_0(v)$, our goal is to determine the tower height that maximizes the probability that wind speed lies within the range $V_{ci} < v < V_r$. If the wind speed frequency distribution and the cumulative distribution functions at any height h are $f_h(v)$ and $F_h(v)$ respectively and if P is the probability that wind speed at that height lies within the range $V_{ci} < v < V_r$ then;

$$P = F_h(v_r) - F_h(v_{ci}) \quad (17),$$

From (16) and (17), P can be expressed as

$$P = F_0(x v_r) - F_0(x v_{ci}) \quad (18)$$

To make P maximum and thus maximizing the duration of the wind speed range $V_{ci} < v < V_r$; dP/dx must equal zero.

$$\frac{dP}{dx} = v_r f_0(x v_r) - v_{ci} f_0(x v_{ci}) \quad (19)$$

Hence,

$$v_r f_0(x v_r) - v_{ci} f_0(x v_{ci}) = 0$$

(20)

Equation (20) can be solved for the values of x for maximum duration in the of wind speed in the range $V_{ci} < v < V_r$. Having x determined, it is straight forward to determine the optimum tower height.

$$h_{opt} = h_0 x^{-1/\alpha} \quad (21)$$

Applying the condition of (20) to the Weibull distribution yields

$$\frac{v_r}{c} k \left(\frac{x v_r}{c}\right)^{k-1} e^{-\left(\frac{x v_r}{c}\right)^k} - \frac{v_{ci}}{c} k \left(\frac{x v_{ci}}{c}\right)^{k-1} e^{-\left(\frac{x v_{ci}}{c}\right)^k} = 0 \quad (22)$$

This can be simplified to

$$\left(\frac{v_r}{v_{ci}}\right)^k = e^{\left(\left(\frac{x}{c}\right)^k (v_r^k - v_{ci}^k)\right)} \quad (23)$$

A closed form solution for the value of x can be obtained as follows.

$$x = \frac{c}{v_{ci}} \left(\frac{k \ln \left(\frac{v_r}{v_{ci}}\right)}{(v_r/v_{ci})^k - 1}\right)^{1/k} \quad (24)$$

3. Results and discussion

In this section, the Weibull parameters of 20 different sites in Egypt are used to determine the optimum tower heights of 9 different WECSs. Data of the 20 sites are taken from [10] and are listed in Table 3, whereas the characteristic wind speeds of the 9 turbines used are listed in Table 4. To check the effect of installing the WECS at the calculated tower height, two different WECSs are used for five different sites in Egypt. To test the ability of the optimum tower to indicate the WECS-site matching, the capacity factor is calculated for each WECS at the calculated optimum tower height. It is to be mentioned that the optimum tower height in the context of this paper means the tower height that maximizes the duration of the effective wind speed range where the WECS output follows the power available in the wind. Other economic and technical factors, although essential for determining the optimum tower height, are not considered in this paper. The main focus of this work is on the use of the optimum tower height as an indicator to the WECS-site matching.

Table III
WSFD parameters for the test sites

No.	Site	c (m/s)	k
1	El-Mathany	6.40	2.33
2	Ras El-Hekma	7.20	2.23
3	El-Galala	6.70	2.41
4	Port Said	5.30	2.32
5	Nuweiba	6.20	2.58
6	Nabq	7.70	2.04
7	Katamaya	6.00	2.66
8	Ras Sedr	8.50	3.06
9	Abu darag	10.1	3.50
10	Zafarana	10.2	3.19
11	Saint paul	9.40	3.25
12	Ras Ghareb	11.0	3.40
13	Gulf of El-Zayt NW	11.8	3.70

14	Gulf of El-Zayt	11.5	3.29
15	Hurgada WETC	7.60	2.32
16	Kosseir	6.50	2.32
17	Kharga	7.40	2.57
18	Dakhla South	7.30	3.31
19	Shark El-Ouinat	7.20	3.29
20	Abu Simbel	6.40	2.76

Table 4.
Data for the WECSs

No.	V_{ci}	V_r	V_{co}
1	4	12.5	25
2	4	14	25
3	3	11.8	25
4	3	11	22
5	3	10.3	22
6	3	9.90	22
7	4	14	25
8	4	15	30
9	5	14	30

The calculated optimum tower heights for the 9 wind turbines at the 20 sites are shown in Fig. 7, where the large variations in the optimum tower height can be noticed. Depending on the wind turbine parameters and the wind speed statistical parameters of the site the optimum tower height varies from about few tens to several hundreds of meters. It can be noticed that some sites, sites 9 to 14, have optimum tower heights much lower than others; the low tower heights indicate that the wind resource at these sites is plentiful and there well is a chance for further increase in the tower height and additional improvement in the WECS capacity factor and economic value. On the contrary, high tower heights at other sites indicate the poor wind energy resource at these sites. It can also be noticed that the optimum tower heights for WECSs 3, 4, 5 and 6 at all sites are lower than those for the other WECSs; this is because these WECSs have their V_{ci} and V_r lower than those for the others. However, for these calculated values of optimum tower heights to have a meaning or to be used as indication for the WECS-site matching, it should be related to some measure of the WECS performance such as the total energy production, or the capacity factor of the WECS.

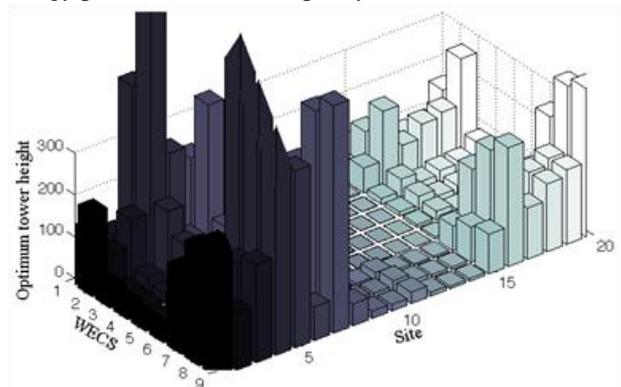


Fig. 7 Optimum tower heights for the 9 WECSs at the 20 sites

The capacity factor is calculated for each of the 9 WECSs installed at the calculated optimum tower height

for each of the 20 sites; the calculated capacity factors are shown in Fig. 8. It can be noticed that, despite the large differences in the values of the optimum tower heights as shown in Fig. 7, the capacity factors are almost all equal to 0.4 or very near to it with a slight deviation. This is a very important feature of the proposed optimum tower height as it can be considered an indicator or a pointer to the height at which the WECS will have a capacity factor of 0.4. If this height is small means that there is chance for additional increase in the tower height with additional increase in the energy gathered and further improvement in the capacity factor of the WECS; this means that the WECS is suitable for use at the site. On the other hand, if the optimum tower height is too high to be practical means that the WECS is not suitable for installation at the site as the practical limits will make the WECS to be installed at a lower height leading to reduction in the energy gathered and a lower capacity factor.

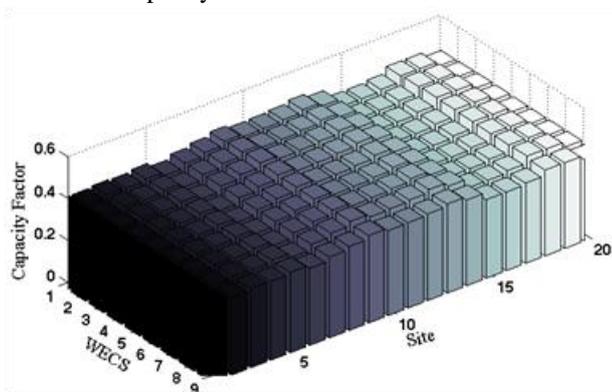


Fig. 8 Capacity factor for the 9 WECSs at the optimum tower heights in the 20 sites

4. CONCLUSION

This paper presents an analytically derived formula for the optimum tower height of a WECS at a given site. This optimum tower height is derived as function of both the wind turbine characteristics speeds and the statistical parameters of the WSFD of the site. The optimum tower height is derived to maximize the probability of the effective wind speed range.

Calculations using the wind data of 20 sites and the data of 9 different sets of wind turbine characteristic speeds

showed that the optimum tower height as defined in this paper ensures a capacity factor for the WECS of not less than 0.4, which makes it a good and fast indicator for the suitability of a given WECS for use in a given site. The smaller this height the better the performance of the WECS, because there will be a room for further increase in tower height and additional improvement in the system capacity factor.

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