

# Thermal Unit Commitment Solution Using an Improved Lagrangian Relaxation

Farid Benhamida, E. N. Abdallah and A. H. Rashed

Department of Electrical Engineering,  
Alexandria University  
Faculty of Engineering, P.O.BOX 21544, Alexandria (Egypt)  
email: bfaridoo@yahoo.fr

**Abstract.** An improved Lagrangian relaxation (LR) solution to the thermal unit commitment problem (UCP) is proposed in this paper. The algorithm is characterized by: (1) a new Matlab function to determine the optimal path of the dual problem, (2) new initialization procedure of Lagrangian multipliers, based on both unit and time interval classification, (3) a flexible adjustment of Lagrangian multipliers, and (4) a dynamic search for uncertain stage scheduling, using a Lagrangian relaxation - dynamic programming method (LR-DP). After the LR best feasible solution is reached, and when identical or similar units exist, a unit decommitment is used to adjust the solution. The proposed algorithm is tested and compared to conventional Lagrangian relaxation (LR), genetic algorithm (GA), evolutionary programming (EP), Lagrangian relaxation and genetic algorithm (LRGA), and genetic algorithm based on unit characteristic classification (GAUC) on systems with the number of generating units in the range of 10 to 100. The total system production cost of the proposed algorithm is less than the others especially for the larger number of generating units. Computational time was found to increase almost linearly with system size, which is favorable for large-scale implementation.

## Key Words

Unit commitment, Generation Scheduling, Lagrangian Relaxation, Unit classification

## List of symbols

$P_i^t$ : output power of unit  $i$  at period  $t$  (MW);  
 $F_i(P_i^t)$ : fuel cost of unit  $i$  when its output power is  $P_i^t$  (\$);  
 $S_i^t$ : start-up price of unit  $i$  at period  $t$  (\$);  
 $u_i^t$ : commitment state of unit  $i$  at period  $t$ , ( $u_i^t=1$ : unit is on-line and  $u_i^t=0$  unit is off-line);  
 $N$ : total number of generating units;  
 $T$ : total number of scheduling periods;  
 $a_b, b_b, c_b$ : Coefficients for the quadratic cost curve of generating unit  $i$ ;  
 $X_{off, i}^t, X_{on, i}^t$ : number of hours the unit has been off-line/on-line (h);  
 $X_i^0$ : Initial condition of a unit  $i$  at  $t=0$ ,  $X_i^0 > 0$ : on-line unit,  $X_i^0 < 0$ : off-line unit (h);  
 $T_i^{up}$  minimum up time (h);  
 $T_i^{down}$  minimum down time (h);  
 $HS^t, CS^t$ : the unit's hot/cold startup cost (\$);

$CH^t$ : is the cold start hour (h);  
 $D^t$ : customers' demand in time interval  $t$ .  
 $R^t$ : The spinning reserve requirements  
 $\lambda^t, \mu^t$ : Lagrangian multipliers at hour  $t$  (\$/MWh);  
 $J, Q$ : primal and dual solution of the LR based UC algorithm;  
 $\varepsilon$ : Pre-specified tolerance.

## 1. Introduction

Unit commitment problem (UCP) is a nonlinear, mixed integer combinatorial optimization problem. It is defined as the problem of how to schedule generators economically in a power system in order to meet the requirements of load and spinning reserve. Usually this problem is considered over some period of time, such as the 24 hours of a day or the 168 hours of a week. It is a difficult problem to solve in which the solution procedures involve the economic dispatch problem as a sub-problem.

Since the problem was introduced, several solution methods have been developed. However, they differ in the solution quality, computational efficiency and the size of the problem they can solve. These methods or approaches have ranged from highly complex and theoretically complicated methods to simplified methods. In the past, various approaches such as DP [1], B&B [2] and Lagrangian relaxation (LR) [3] were proposed for solving the UCP. However, not all of these methods are regarded as feasible and/or practical as the size of the system increases.

For moderately sized production systems, exact methods, such as dynamic programming (DP) or branch-and-bound (B&B) [2] can be used to solve the UCP, successfully. For larger systems, exact methods fail because the size of the solution space increases exponentially with the number of time periods and units in the system. As a result, the computation time of exact methods becomes impractical. In these cases heuristic methods (evolutionary programming (EP), Tabu Search (TS), Simulated Annealing (SA), Genetic Algorithms (GA), etc) can be used to produce near optimal solutions in a

reasonable computation time. For heuristic methods optimality is not given such a high priority but the emphasis is on finding good solutions in a short time. This often results in the solution method being more simple and transparent than exact solution methods [4].

The application of LR in the scheduling of power generations was proposed in the late 1970s. These earlier methods used LR to substitute the common linear programming (LP) relaxation approach as a lower bound in the B&B technique [5]. In this regard, great improvement of computational efficiency was achieved compared with previous B&B algorithms.

In recent years, methods based on LR, have become the most dominant ones. This approach has shown some potential in dealing with systems that consist of hundreds of generating units and is motivated by the separable nature of the problem, and several examples have been reported in the literature.

Based on the sharp bound provided by the Lagrangian dual optimum, it is expected that a suboptimal feasible solution near the dual optimal point can be accepted as a proper solution for the primal problem. A more direct and fairly efficient methodology which has used this idea was presented in [6] by Merlin, for UCP using LR method and validated at Electricite De France. Due to its reasonable performance, the successive improvement of the LR algorithm, in the last few years, has mainly followed the work in [6]. The problem which is supposed to be handled by this algorithm consists of thermal units only.

In [7], they combined LR, sequential UC based on the least reserve cost index and unit decommitment (UD) based on the highest average spinning reserve cost index. However, this method could not decommit some units that violate the minimum up time constraints even though the excessive reserve exists, leading to a higher production cost.

In the advent of heuristic approaches, GA [8], EP [9], SA [10], and TS [11] have been proposed to solve the UC problems. Nevertheless, the obtained results by GA, EP, and SA required a considerable amount of computational time especially for a large system size. There was an attempt to combine the LR and GA method (LRGA) to obtain a higher quality of UC solution in a shorter time by using normalized Lagrange multipliers as the encoded parameter [12].

## 2. Unit Commitment Problem Formulation

The objective of the UCP is to minimize the system operating costs, which is the sum of production and start-up costs of all units over the entire study time span (e.g., 24 h), under the generator operational and spinning reserve constraints. Mathematically, the objective function, or the total operating cost of the system can be written as follows:

$$J = \min_{P_i^t, u_i^t} f(P_i^t, u_i^t) =$$

$$\min_{P_i^t, u_i^t} \left( \sum_{t=1}^T \sum_{i=1}^N u_i^t [F_i(P_i^t) + S_i^t(1-u_i^{t-1})] \right) \quad (1)$$

Subject to:

(1) The start-up cost is modeled by the following function of the form:

$$S_i^t = \begin{cases} HS^i, & \text{if } X_{off,i}^t \leq T_i^{down} + CH^i \\ CS^i, & \text{if } X_{off,i}^t > T_i^{down} + CH^i \end{cases} \quad (2)$$

(2) Power balance

$$\sum_i^N u_i^t P_i^t = D^t \quad (3)$$

(3) Spinning reserve requirements:

$$\sum_i^N u_i^t P_i^{\max} \geq D^t + R^t \quad (4)$$

Generating limits:

$$u_i^t P_i^{\min} \leq P_i^t \leq u_i^t P_i^{\max} \quad (5)$$

Minimum up time constraint:

$$(X_{on,i}^{t-1} - T_i^{up})(u_i^{t-1} - u_i^t) \geq 0 \quad (6)$$

$$X_{on,i}^t = (X_{on,i}^{t-1} + 1) u_i^t \quad (7)$$

Minimum down time constraint:

$$(X_{off,i}^{t-1} - T_i^{down})(u_i^t - u_i^{t-1}) \geq 0 \quad (8)$$

$$X_{off,i}^t = (X_{off,i}^{t-1} + 1)(1 - u_i^t) \quad (9)$$

Fuel cost functions  $F_i(P_i^t)$  is frequently represented by the polynomial function:

$$F_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)^2 \quad (10)$$

## 3. An Improved Flexible Lagrangian Relaxation Technique

In the Lagrangian relaxation approach, the system operating cost function of (1) of the unit-commitment problem is related to the power balance and the spinning reserve constraints via two sets of Lagrangian multipliers to form a Lagrangian dual function.

$$L(P, u, \lambda, \mu) = f(P, u) + \sum_{t=1}^T \lambda^t \left( D^t - \sum_{i=1}^N u_i^t P_i^t \right) + \sum_{t=1}^T \mu^t \left( D^t + R^t - \sum_{i=1}^N u_i^t P_i^{\max} \right) \quad (11)$$

The LR procedure solves the UC problem through the dual problem optimization procedure attempting to reach the constrained optimum.

The dual procedure attempts to maximize the Lagrangian with respect to the Lagrangian multipliers  $\lambda^t$  and  $\mu^t$ , while minimizing it with respect to the other variables  $P_i^t, u_i^t$  subject to the unit constraints in (5) through (9). The dual

problem is thus the search of the dual solution ( $Q$ ) expressed as:

$$Q = \max(\min_{\lambda, \mu} L(P, u, \lambda, \mu)), \quad \lambda^t \geq 0 \text{ and } \mu^t \geq 0 \quad (12)$$

The Lagrangian function of (11) is rewritten as

$$L(P, u, \lambda, \mu) = f(P, u) - \sum_{t=1}^T \lambda^t \sum_{i=1}^N u_i^t P_i^t - \sum_{t=1}^T \mu^t \sum_{i=1}^N u_i^t P_i^{\max} + \sum_{t=1}^T \lambda^t D^t + \sum_{t=1}^T \mu^t (D^t + R^t) \quad (13)$$

When the Lagrangian multipliers  $\lambda^{t(k)}$  and  $\mu^{t(k)}$  are fixed for iteration  $k$ , the last two terms of the Lagrangian in (13) are constant and can be dropped from the minimization problem. Hence, the system (coupling) constraints can be relaxed and the search for the dual solution can be done through the minimization of the Lagrangian as:

$$\min_{P_i^t, u_i^t} L(P, u, \lambda^{(k)}, \mu^{(k)}) = \min_{P_i^t, u_i^t} \sum_{t=1}^T \sum_{i=1}^N u_i^t \{ F_i(P_i^t) + S_i^t (1 - u_i^{t-1}) - \lambda^{t(k)} P_i^t - \mu^{t(k)} P_i^{\max} \} \quad (14)$$

Then, the minimum of the Lagrangian function is solved for each generating unit over the time horizon, that is

$$\min_{P_i^t, u_i^t} L(P, u, \lambda^{(k)}, \mu^{(k)}) = \sum_{t=1}^T \min_{P_i^t, u_i^t} \sum_{i=1}^N u_i^t \{ F_i(P_i^t) + S_i^t (1 - u_i^{t-1}) - \lambda^{t(k)} P_i^t - \mu^{t(k)} P_i^{\max} \} \quad (15)$$

Subject to constraints in (5) through (9).

#### A. The Dual problem optimization

In the Lagrangian relaxation method, the dual solution is obtained for each unit separately.

When the state  $u_i^t = 0$ , the value of the function to be minimized is equal zero (the unit is off-line).

When the state  $u_i^t = 1$ , the value to be minimized is:  $F_i(P_i^t) - \lambda^{t(k)} P_i^t$  (16)

The startup cost and the last term in (15) are dropped since the minimization is with respect to  $P_i^t$ .

When the units' fuel cost functions are represented as polynomial functions as in (10), the minimum of (16) can be found by taking its first derivative.

$$d(F_i(P_i^t) - \lambda^t P_i^t) / dP_i^t = dF_i(P_i^t) / dP_i^t - \lambda^t = 0 \quad (17)$$

$$\text{Hence, } P_i^{t(k)} = (\lambda^{t(k)} - b_i) / 2c_i \quad (18)$$

$$\bullet \text{ If } P_i^{t(k)} < P_i^{\min} \text{ then } P_i^{t(k)} = P_i^{\min} \quad (19)$$

$$\bullet \text{ If } P_i^{t(k)} > P_i^{\max} \text{ then } P_i^{t(k)} = P_i^{\max} \quad (20)$$

For known  $\lambda^{t(k)}$ , then  $P_i^{t(k)}$  is obtained by (18) through (20).

#### 1). A new Matlab function to determine the optimal path

To minimize the term in (15) for each unit, over the scheduled time  $T$ , subject to minimum up and down time constraints in (6) through (9), DP is often used to determine the optimal schedule. Dynamic programming CPU time increases at least linearly with  $N$  and  $T$  (upper bounded by  $N [4(T-1) + 2]$  additions and  $2N(T-1)$  comparisons), [3] and [13].

A reduction of the search domain, which is defined by  $2^T$  combinations, can be made by discarding the infeasible combinations from the domain. The optimal combination which minimize (15) for a unit  $i$  can be determined by direct evaluation of all feasible combinations. A Matlab function is developed for this purpose.

This function gives all feasible combinations ( $m_i$ ) of unit  $i$  over the scheduling period  $T$  which satisfy the minimum up and down time constraints given its initial state and condition.

Function input:  $X_i^0, T, T_i^{up}, T_i^{down}$ .

Function output:  $[u_{i,j}^t]$  which is a  $(T \times m_i)$  matrix containing all feasible combination as

$$[u_{i,j}^t] = \begin{pmatrix} u_{i,1}^1 & u_{i,2}^1 & \dots & u_{i,m_i}^1 \\ u_{i,1}^2 & u_{i,2}^2 & \dots & u_{i,m_i}^2 \\ \vdots & \vdots & & \vdots \\ u_{i,1}^T & u_{i,2}^T & \dots & u_{i,m_i}^T \end{pmatrix}, \quad i = 1, \dots, N; \quad (21)$$

Then the optimal solution  $P_i^{(k)}$  and its corresponding path  $u_{i,j}$  (combination), given the Lagrangian multipliers  $\lambda^{(k)}, \mu^{(k)}$  of iteration  $k$ , is obtained by the following procedure:

Step 1: Running the Matlab function to obtain all feasible combinations ( $m_i$ ) of unit  $i$  over the scheduling period  $T$  which satisfies the minimum up and down time constraints given its initial state and condition.

Step 2: For each  $u_{i,j}, j = 1, \dots, m_i, t = 1, \dots, T$  calculate the contribution term of unit  $i$  in a specific period  $t$  using the following equation

$$u_{i,j}^t \{ F_i(P_i^{t(k)}) + S_i^t (1 - u_{i,j}^{t-1}) - \lambda^{t(k)} P_i^{t(k)} - \mu^{t(k)} P_i^{\max} \} \quad j = 1, \dots, m_i \quad t = 1, \dots, T \quad (22)$$

Step 3: For each  $u_{i,j}, j = 1, \dots, m_i$ , calculate the contribution term which correspond to unit  $i$  over the total period  $T$  using the following equation:

$$\sum_{t=1}^T [u_{i,j}^t \{ F_i(P_i^{t(k)}) + S_i^t (1 - u_{i,j}^{t-1}) - \lambda^{t(k)} P_i^{t(k)} - \mu^{t(k)} P_i^{\max} \}] \quad j = 1, \dots, m_i \quad (23)$$

Step 4: Obtain optimal solution  $P_i^{(k)}$  and its corresponding path  $u_{i,j}$  (combination), by taking the least valued contribution terms obtained in step 3. We have not to check the path vis-à-vis the minimum up and down time since it is a feasible one.

Step 5: repeat step 1 to step 4 for all units to obtain  $P^{(k)}, u^{(k)}$ .

The values of the system variables  $P^{(k)}, u^{(k)}, \lambda^{(k)}, \mu^{(k)}$  are substituted back into the Lagrangian (11),  $L(P^{(k)}, u^{(k)}, \lambda^{(k)}, \mu^{(k)})$  to determine the dual solution  $Q^{(k)}$ :

$$L(P^{(k)}, u^{(k)}, \lambda^{(k)}, \mu^{(k)}) = f(P_i^{t(k)}, u_i^{t(k)}) + \sum_{t=1}^T \lambda^{t(k)} \left( D^t - \sum_{i=1}^N u_i^{t(k)} P_i^{t(k)} \right) + \sum_{t=1}^T \mu^{t(k)} \left( D^t + R^t - \sum_{i=1}^N u_i^{t(k)} P_i^{\max} \right) \quad (24)$$

Provided that the dual solution is feasible with respect to the spinning reserve constraint (4) and the following constraint regarding the minimum output power of the scheduled units is satisfied:

$$\sum_{i=1}^N u_i^t P_i^{\min} \leq D^t \quad \forall t = 1, \dots, T \quad (25)$$

The inequalities related to the spinning reserve constraints (4) do not impose an upper bound on the amount of reserve. However, common sense for an economic schedule indicates that there should not be too much excess MW reserve because it would certainly increase the cost associated with the corresponding dual solution. Therefore, in the searching algorithm, a slack term ( $s^t$ ) is included in the reserve constraint to assess the quality of the dual solution. The upper-bound limit introduced by the slack term restricts the solution space and therefore may prevent the optimal solution to be found. In addition, the value of the slack term may affect the convergence of the process. Unfortunately, there is no mathematical guideline for properly selecting the value of slack term ( $s^t$ ) [13].

Hence, in the searching algorithm, the following constraints are included implicitly to test the validity of the commitment schedule.

$$D^t + R^t \leq \sum_i u_i^t P_i^{\max} \leq D^t + R^t + s^t \quad (26)$$

In this paper  $s^t$  is specified using a new heuristic algorithm based on both unit and time interval classification.

### B. A new initial scheduling of UC

The initial values of Lagrangian multipliers are very critical to the LR solution since they may prevent LR from reaching the optimal solution or require a longer computational time to reach one [14]. Different initial values may also lead LR to different solutions. In [15], the initial multiplier  $\lambda^t$  was set to the hourly system marginal cost of the schedule to satisfy the power balance constraint and the initial multiplier  $\mu^t$  was set to zero, leading to an infeasible initial solution. Alternatively, the initial multiplier  $\lambda^t$  was set to the hourly system marginal

cost of the schedule to satisfy both the power balance and spinning reserve constraint, whereas the initial multiplier  $\mu^t$  was set to zero which was generally lower than the optimal value [16].

An initialization procedure which intends to create a high quality feasible schedule in the first iteration is described here, based on unit and time interval classification.

#### 1). Unit Classification

In general, generation units can be classified into three types: *base load* units with low operation cost  $F_i$ , high startup cost  $S_i$ , and long minimum up/down times  $T_i^{up}, T_i^{down}$ ; *intermediate load* units with medium operating cost, medium startup cost and medium minimum up/down time, and *peak load* units with high operation cost, low startup cost and short minimum up/down time. Base load units should not be shut down. In other words they constitute the must run constraint. Intermediate load units could be committed during on-peak and decommitted during off-peak periods. Finally, peak load units could be frequently turned on and off.

Following this classification, the  $N$  units of an  $N$ -unit system can be classified into a set  $N_b$  of base load units, a set  $N_I$  of intermediate load units and a set  $N_p$  of peak load units according to unit full load average production costs ( $flac$ ) and unit operational constraints where:

$$\text{Where } flac = F(P_i^{\max})/P_i^{\max} \quad (27)$$

#### 2). Time interval classification

The overall study period is decomposed into several interval classes as follow:

(1)  $T_{bd}$  presents the set of scheduling intervals  $t$  where  $t \in T_b$  and the upper-bound limit of the spinning reserve is satisfied:  $\sum_{i \in N_b} P_i^{\max} - D^t - R^t \leq s^t$ ,  $T_b$  being the set of

scheduling intervals  $t$  where base units can produce enough power to satisfy the inequality  $\sum_{i \in N_b} P_i^{\max} \geq D^t + R^t$ . Hence, during the

intervals ( $T_{bd}$ ) only base units are committed.

(2)  $T_{Id}$  presents the set of scheduling intervals  $t$  where  $t \in T_I \cap T_{I0}$ , and in which  $\sum_{i \in N_b \cup N_I} P_i^{\max} - D^t - R^t \leq s^t$ .

Here  $T_I$  presents the set of scheduling intervals  $t$  where the group of base and intermediate units cannot produce enough power to satisfy the constraint

$$\sum_{i \in N_b \cup N_I} P_i^{\max} \leq D^t + R^t + \min_{i \in N_I} (P_i^{\max}), \text{ while } T_{I0} \text{ presents}$$

the set of scheduling hour's  $t$  where the base and intermediate units grouped can produce enough power to satisfy the spinning reserve

$$\text{constraint } \sum_{i \in N_b \cup N_I} P_i^{\max} \geq D^t + R^t. \text{ At these intervals } (T_{Id})$$

both base and intermediate units are committed. Note that  $T_b \subset T_{I0}$ .

(3)  $T_I - T_{Id}$  give the set of scheduling hour's  $t$ , where peak units must be committed, for these scheduling periods the peak units are selected one by one, based on the *flac*, until enough capacity is reached to fulfill the spinning reserve constraints.

(4)  $T_{I0} - T_{Id} - T_{bd}$  give the set of scheduling hour's  $t$ , where intermediate units must be committed, for these scheduling periods intermediate units are selected one by one, based on the *flac*, until enough capacity is reached to fulfill the spinning reserve constraints.

The slack term ( $s^t$ ) is defined as

$$\text{If } t \in T_b, s^t = \alpha \times \max_{i \in N_b} (P_i^{\max}) \quad (28)$$

$$\text{If } t \in T_I, s^t = \alpha \times \max_{i \in N_p} (P_i^{\max}) \quad (29)$$

$$\text{If } t \in T_{I0} \text{ and } t \notin T_b, s^t = \alpha \times \max_{i \in N_I} (P_i^{\max}) \quad (30)$$

Where  $\alpha$  is a tuning constant. Through the application of the method, its most suitable value was found to be 2.

Table 1 gives the initial commitment states of different sets  $N_b$ ,  $N_I$  and  $N_p$  during different time interval classes.

Table 1. Initial scheduling of UCP based on unit and time interval classification

Time set	Unit Sets		
	$N_b$	$N_I$	$N_p$
$t \in T_{bd}$	$u_i^t = 1$	$u_i^t = 0$	$u_i^t = 0$
$t \in T_{Id}$	$u_i^t = 1$	$u_i^t = 1$	$u_i^t = 0$
$t \in T_I - T_{Id}$	$u_i^t = 1$	$u_i^t = 1$	$u_i^t$ initialized based on <i>flac</i>
$t \in T_{I0} - T_{Id} - T_{bd}$	$u_i^t = 1$	$u_i^t$ initialized based on <i>flac</i>	$u_i^t = 0$

### 3). Initial value of Lagrangian multipliers

The initial value of Lagrangian multipliers  $\lambda^{(0)}$  are set as follow:

(1) For each hour  $t \in T_{I0} - T_{Id} - T_{bd}$  and  $t \in T_I - T_{Id}$ , the group of identical units with the least (*flac*) will be committed one group by one group until the spinning reserves constraint is satisfied as shown in Table 1. Subsequently, economic dispatch in each hour is carried out to obtain the hourly equal lambda which is initially set to Lagrangian multipliers  $\lambda^{(0)}$ .

(2) For each  $t \in T_{bd} \cup T_{Id}$ , as at these period a predefined UC is established as shown in Table 1. Lagrangian multipliers  $\lambda^{(0)}$  are set to the hourly equal lambda, after running an economic dispatch program for these periods. The initial value of each non-negative Lagrangian multipliers  $\mu^{(0)}$  is set as follow:

$$\mu^{(0)} = \max \left( \max_{i=1, \dots, M} \left( \frac{1}{P_i^{\max}} (F_i(P_i^t) + \frac{S_i^t}{T_i^{up}} - \lambda^{(0)} P_i^t) \right), 0 \right) \quad (31)$$

Where  $M$  is the marginal unit with the highest (*flac*), giving the sufficient spinning reserve at hour  $t$ .

### C. Updating of the Lagrangian Multiplier

In general, adjusting Lagrangian multiplier by sub-gradient method is not efficient in the presence of the spinning reserve constraint [6]; one of the shortcomings of this method is the slow convergence. The LR performance is heavily dependent on the method used to update the multipliers. In this paper, a flexible sub-gradient rule is proposed to update the Lagrangian multiplier and designed such that the step size is large at the beginning of iterations and smaller as the iteration grows. Each nonnegative  $\lambda^t$  and  $\mu^t$  are adaptively updated by,

$$\lambda^{t(k)} = \max \left( 0, \lambda^{t(k-1)} + \frac{P_M^t}{(\rho + \theta \times k) \times \text{norm}(P_M^t)} \right) \quad (32)$$

$$\mu^{t(k)} = \max \left( \mu^{t(k-1)} + \frac{SR_M^t}{(\rho + \theta \times k) \times \text{norm}(SR_M^t)}, 0 \right) \quad (33)$$

Where

$$P_M^t = D^t - \sum_{i=1}^N u_i^t P_i^t \quad (34)$$

$$SR_M^t = D^t + R^t - \sum_{i=1}^N u_i^t P_i^{\max} \quad (35)$$

$$\text{norm}(P_M^t) = \sqrt{(P_M^1)^2 + (P_M^2)^2 + \dots + (P_M^T)^2} \quad (36)$$

$$\text{norm}(SR_M^t) = \sqrt{(SR_M^1)^2 + (SR_M^2)^2 + \dots + (SR_M^T)^2} \quad (37)$$

The values of  $\rho$  and  $\theta$  are divided into four cases depending on the signs of  $P_M^t$  and  $SR_M^t$ .

Case 1)  $P_M^t \geq 0$  and  $SR_M^t \geq 0$ : updating both  $\lambda^t$  and  $\mu^t$  by using  $\rho = 0.03$  and  $\theta = 0.06$ .

Case 2)  $P_M^t < 0$  and  $SR_M^t < 0$ : updating both  $\lambda^t$  and  $\mu^t$  by using  $\rho = 0.5$  and  $\theta = 0.3$ .

Case 3)  $P_M^t < 0$  and  $SR_M^t > 0$ : updating only  $\mu^t$  by using  $\rho = 0.03$  and  $\theta = 0.06$ .

Case 4)  $P_M^t > 0$  and  $SR_M^t < 0$ : updating only  $\lambda^t$  by using  $\rho = 0.5$  and  $\theta = 0.3$ .

The general guidelines for selecting their values are explained in [17].

In fact, updating the two multipliers  $\lambda^t$  and  $\mu^t$  in hour  $t$  must move them in the same direction. In hour  $t$ , if  $P_M^t$  and  $SR_M^t$  have the same signs, either positive or negative,  $\lambda^t$  and  $\mu^t$  will be updated (increase or decrease) by (32) and (33), respectively.

When the total dual generation output is larger than the load in that hour ( $P_M^t < 0$ ) but the spinning reserve is insufficient ( $SR_M^t > 0$ ), more committed unit(s) are required to satisfy the spinning reserve constraints. However, updating  $\lambda^t$  by (32) will decrease its value,

resulting in committing less units. Therefore, when  $(P_M^t < 0)$  and  $(SR_M^t > 0)$ , only  $\mu^t$  will be updated.

On the contrary, when the spinning reserve is sufficient  $(SR_M^t < 0)$ , but the total dual generation output is less than the load in that hour  $(SR_M^t > 0)$ , updating  $\mu^t$  by (33) will decrease its value, resulting in committing less units. Therefore, when  $(P_M^t > 0)$  and  $(SR_M^t < 0)$ , only  $\lambda^t$  will be updated.

Note that the sub-gradient method generally needs a large number of iterations to converge to near the dual optimum [17]. The proposed flexible sub-gradient method using high-quality initial feasible multipliers proved to require much lower number of iterations to converge, leading to much less computational time.

#### D. Dynamic Economic Dispatch (DED) [18]

To replace conventional economic dispatch algorithm, a more accurate and flexible problem formulation of DED is developed to facilitate the interaction with UC schedule, The DED solver use the Hopfield Neural Network, which make it a very fast solver and suitable to UCP.

If the 24-h schedule is feasible at iteration  $k$ , a DED is carried out to determine the optimal generation power outputs for each of the 24 h, and the total production cost  $J^{(k)}$ .

#### E. Checking for Convergence

The convergence of the proposed LR-UC algorithm can be measured by the relative duality gap between the primal and dual solutions.

$$\text{Relative duality gap} = \left( (J^{(k)} - Q^{(k)}) / Q^{(k)} \right) \times 100 \quad (38)$$

The process stops when the relative duality gap is smaller than a pre-specified tolerance  $\mathcal{E}$ , or when a pre-specified maximum number of iterations is reached.

The sensitivity of the integer variables corresponding to the generating unit statuses ( $u_i^t$ ) to small adjustments in the Lagrangian multipliers may cause the algorithm to oscillate around the optimal solution. As such, there is no guarantee that the solution achieved in the last iteration of the iterative process will be feasible or optimal. Hence, in the computational model developed in the paper, a running record of the feasible solutions is kept so that the final solution is the one corresponding to the most economical schedule, i.e., the one with the minimum primal solution ( $J$ ).

### 4. Identical Unit Decommitment

When identical or similar units exist the LR could find only sub-optimal solutions [14]. These units have the identical cost parameters  $a_i$ ,  $b_i$ ,  $c_i$ , and startup cost which will be simultaneously committed or decommitted. This will not lead to the optimal solution because committing

one unit at a time will be less expensive than committing a whole group of units, which may lead to over commitment. Thus, after committing a group of identical units, a unit of which is decommitted one at a time if it does not violate the minimum up time constraint until either the spinning reserve requirement is not satisfied or there is only one unit left. The identical unit decommitment procedure is as follows:

Step 1) Get the initial feasible solution  $[u_i^t]$ ,  $i = 1, \dots, N$ ,  $t = 1, \dots, T$ .

Step 2) Calculate the excess spinning reserve of every hours,

$$R_{ex}^t = \sum_i^N u_i^t P_i^{\max} - D^t + R^t \quad (39)$$

Step 3) Initialize  $t = 1$

Step 4) Initialize  $i = 1$

Step 5) If the excess spinning reserve  $R_{ex}^t$  is greater than the maximum generation of unit  $i$ , and this unit is already committed, check if decommitting the unit would violate its minimum up time constraints. Decommit the unit  $i$ ,

$$\text{If, } X_{on,i}^t = 1, \text{ and } X_{on,i}^{t+T_i^{up}} = T_i^{up} + 1 \quad (40)$$

$$\text{or if } X_{on,i}^t > T_i^{up}, \text{ and } X_{off,i}^{t+1} = 1 \quad (41)$$

$$\text{or if } X_{on,i}^t = 1, \text{ and } \sum_{k=t}^T u_i^k = T - k + 1 \quad (42)$$

$$\text{or if } T_i^{up} = 1 \quad (43)$$

Otherwise, let the unit committed.

Step 6) If  $t = T$  stop, else go to step 7

Step 7) Update  $[u_i^t]$  and  $R_{ex}^t$ , replace  $i$  by  $i + 1$ ,

Step 8) If  $i = N$ , replace  $t$  by  $t + 1$ , and go to step 4. Otherwise, go to step 5.

### 5. Numerical Results

A 10-unit system [8] is selected as a test system. System data and load demand are given in Tables 2 and 3. The spinning reserve is assumed to be 10% of the demand. The 20, 40, 60, 80, and 100 unit systems are obtained by duplicating the 10-unit base case, whereas the load demand are adjusted in proportion to the system size. The proposed LRUC uses the developed Matlab function to determine the optimal path. A maximum allowable number of 50 iterations was set as a stopping criteria.

Table 2. Unit data of the 10-unit 24 hour test system

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
$P^{\max}$ (MW)	455	455	130	130	162
$P^{\min}$ (MW)	150	150	20	20	25
$a$ (\$/h)	1000	970	700	680	450
$b$ (\$/MWh)	16.19	17.26	16.60	16.50	19.70
$c$ (\$/MW <sup>2</sup> h)	0.00048	0.00031	0.0020	0.00211	0.00398
$T_i^{up}$ (h)	8	8	5	5	6
$T_i^{down}$ (h)	8	8	5	5	6
$HS$	4500	5000	550	560	900
$CS$	9000	10000	1100	1120	1800
$CH$	5	5	4	4	4
$X_i^0$	8	8	-5	-5	-6
$flac$	18.61	19.53	22.24	22.01	23.12

	Unit 6	Unit 7	Unit 8	Unit 9	Unit10
$P^{max}$ (MW)	80	85	55	55	55
$P^{min}$ (MW)	20	25	10	10	10
$a$ (\$/h)	370	480	660	665	670
$b$ (\$/MWh)	22.26	27.74	25.92	27.27	27.79
$c$ (\$/MW <sup>2</sup> h)	0.00712	0.00079	0.00413	0.00222	0.00173
$T_i^{up}$ (h)	3	3	1	1	1
$T_i^{down}$ (h)	3	3	1	1	1
HS	170	260	30	30	30
CS	340	520	60	60	60
CH	2	2	0	0	0
$X_i^0$	-3	-3	-1	-1	-1
flac	27.45	33.45	38.14	39.48	40.06

Table 3. Demand of 10 unit 24 hour test system

Hour	Load (MW)	Hour	Load (MW)	Hour	Load (MW)
1	700	9	1300	17	1000
2	750	10	1400	18	1100
3	850	11	1450	19	1200
4	950	12	1500	20	1400
5	1000	13	1400	21	1300
6	1100	14	1300	22	1100
7	1150	15	1200	23	900
8	1200	16	1050	24	800

#### A. An improvement to the method

The behavior of the units during the iterative search of the LR based solution and the preliminary schedule itself is assessed to define the uncertain intervals, in which commitment states of some units are not certain. In this example these stages are [22, 23]. Then, a dynamic search is performed at these stages, using a DP solution to UC combined with LR (LR-DP), as shown in figure 2, where all possible and feasible paths with respect to minimum up and down time constraints are shown. The optimum path is distinguished by bold lines. The UC solution schedule using the proposed Lagrangian Relaxation combined to DP is shown in table 5.

Table 4 shows simulation results (production costs) obtained by the proposed LR method compared with results obtained by LR [8], GA [8], EP [19], and the combined LRGGA [20] and DPLR [21] methods. Table 6 shows the simulation time obtained by the proposed LR method which is carried out on Pentium M 1.73 GHz processor. Because simulations were carried out on different types of computers, simulation times are not compared. It can be seen that the results of the proposed method is better than other methods in term of total production cost. It can be seen that computational time increases almost linearly with system size.

## 6. Conclusion

This paper presents a Lagrangian relaxation solution to the thermal UCP. An initialization procedure intends to create a high quality feasible schedule in the first iteration is proposed, based on unit and time interval classification. The proposed LR is efficiently and effectively implemented to solve the UC problem. The proposed LR total production costs over the scheduled time horizon are less than conventional LR, GA, EP, LRGGA, and GAUC especially for the larger number of generating units. Moreover, the proposed LR CPU times increase almost linearly with the system size, which is favorable for large-scale implementation.

## References

- [1] W. J. Hobbs, et. al., "An enhanced dynamic programming approach for unit commitment," *IEEE Trans. Power Syst.*, Vol. 3, No. 3, pp. 1201-1205, August 1988.
- [2] C. L. Chen and S. C. Wang, "Branch-and-Bound Scheduling for Thermal Generating Units," *IEEE Trans. on Energy Conversion*, Vol. 8, No. 2, pp. 184-189, June 1993.
- [3] F. Zhuang and F.D. Galiana, "Towards a more rigorous and practical unit commitment by Lagrangian relaxation," *IEEE Trans. Power Syst.*, Vol. 3, No. 2, pp. 763-773, May 1988.
- [4] S. Matsuda et. al., "The representation of large numbers in neural networks and its application to economical load dispatching of electric power," *ICNN*, Vol. 1, pp. 587-592, 1989.
- [5] G. S. Lauer, et. al., "Solution of large-scale optimal unit commitment problems," *IEEE Trans. Power Apparatus Syst.* PAS-101 (1, 1982), pp. 79-86.
- [6] A. Merlin and P. Sandrin, "A New Method for the Unit Commitment at the Electricité de France," *IEEE Trans. Power Apparatus Syst.*, Vol. PAS-102, No. 5, pp. 1218-1225, 1983.
- [7] C. Li, R. B. Johnson, A. J. Svoboda, C. Tseng, and E. Hsu, "A robust unit commitment algorithm for hydro-thermal optimization," *IEEE Trans. Power Syst.*, Vol. 13, No. 3, pp. 1051-1056, Aug. 1998.
- [8] S. A. Kazarlis, A. G. Bakirtzis, and V. Petridis, "A genetic algorithm solution to the unit commitment problem," *IEEE Trans. Power Syst.*, Vol. 11, No. 1, pp. 83-92, Feb. 1996.
- [9] P. Attaviriyanupap, H. Kita, E. Tanaka and J. Hasegawa, "Evolutionary Programming based Dynamic Economic Dispatch," in *Joint Conference Record, The Hokkaido Chapter of The Institute of Electrical and Information Engineers*, Japan, pp. 74-75, 2000.
- [10] A. H. Mantawy, et. al., "A Simulated Annealing Algorithm for Unit Commitment", *IEEE Trans. Power Syst.*, Vol. 13, No. 1, pp. 197-204, Feb. 1998.
- [11] A. H. Mantawy et. al., "Unit Commitment by Tabu Search", *IEE Proc. -Gener. Distrib.*, Vol 145, No. 1, pp. 56-64, Jan. 1998.
- [12] K. Aoki, T. Satoh, M. Itoh, T. Ichimori, and K. Masegi, "Unit Commitment in a Large-Scale Power System Including Fuel Constrained Thermal and Pumping- Storage Hydro," *IEEE Trans. Power Syst.*, Vol. PWR-2, No. 4, pp. 1077-1084, Nov. 1987.
- [13] S. K. Tong and S. M. Shahidepour, "Combination of lagrangian-relaxation and linear programming approaches for fuel-constrained unit-commitment problems," *IEE Proceedings-C*, Vol. 136, No. 3, pp. 162-174, May 1989.
- [14] S. Dekrajangpetch, G. B. Sheble, and A. J. Conejo, "Auction implementation problems using Lagrangian relaxation," *IEEE Trans. Power Syst.*, Vol. 14, No. 1, pp. 82-88, Feb. 1999.
- [15] K. H. Abdul-Rahman, et. al., "A practical resource scheduling with OPF constraints," *IEEE Trans. Power Syst.*, Vol 11, No. 1, pp. 254-259, Feb 1995.
- [16] H. Yan, P. B. Luh, X. Guan, and P. M. Rogan, "Scheduling of hydrothermal power systems," *IEEE Trans. Power Syst.*, Vol. 8, pp. 1358-1365, Aug. 1993.
- [17] M. L. Fisher, "The Lagrangian Relaxation Method for Solving integer Programming Problems," *Management Science*, Vol. 27, No.1, pp. 1-18, Jan. 1981.
- [18] F. Benhamida, E. N. Abdallah, A. H. Rashed, "Solving the Dynamic Economic Dispatch as Part of Unit commitment problem," *AMSE, Modeling General Physics and Electrical Applications*, Vol. 78, No. 5, pp. 49-63, 2005.

[19] K. A. Juste, H. Kita, E. Tanaka, and J. Hasegawa, "An evolutionary programming solution to the unit commitment problem," *IEEE Trans. Power Syst.*, Vol. 14, pp. 1452–1459, Nov. 1999.

[20] C. C. Ping, L. C. Wen, and L. C. Chang, "Unit Commitment by Lagrangian Relaxation and Genetic Algorithms," *IEEE Trans. Power Syst.*, Vol. 15, No. 2, pp. 707-714, May 2000.

[21] T. Senjyu, H. Yamashiro, K. Uezato, and T. Funabashi, "A unit commitment problem by using genetic algorithm based on characteristic classification," in *Proc. IEEE/Power Eng. Soc. Winter Meet.*, Vol. 1, pp. 58-63, 2002.

Table 4. Comparison of total production costs

METHOD	COST(\$)					
	No of units					
	10	20	40	60	80	100
LR [8]	565,825	1,130,660	2,258,503	3,394,066	4,526,022	5,657,277
GA [8]	565,825	1,126,243	2,251,911	3,376,625	4,504,933	5,627,437
EP [19]	564,551	1,125,494	2,249,093	3,371,611	4,498,479	5,623,885
LRGA [20]	564,800	1,122,622	2,242,178	3,371,079	4,501,844	5,613,127
DPLR [21]	564,049	1,128,098	2,256,195	3,384,293	4,512,391	5,640,488
GAUC [21]	563,977	1,125,516	2,249,715	3,375,063	4,505,614	5,640,488
Proposed LR	563937.69	1,122,637	2,243,245	3,363,376	4,484,915	5,604,470

Table 5. Solution of 10 – unit 24-hour using the proposed LRUC method

Hour	Unit Number									
	1	2	3	4	5	6	7	8	9	10
1	1	1	0	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0	0	0
3	1	1	0	0	1	0	0	0	0	0
4	1	1	0	0	1	0	0	0	0	0
5	1	1	0	1	1	0	0	0	0	0
6	1	1	1	1	1	0	0	0	0	0
7	1	1	1	1	1	0	0	0	0	0
8	1	1	1	1	1	0	0	0	0	0
9	1	1	1	1	1	1	1	0	0	0
10	1	1	1	1	1	1	1	1	0	0
11	1	1	1	1	1	1	1	1	1	0
12	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	0	0
14	1	1	1	1	1	1	1	0	0	0
15	1	1	1	1	1	0	0	0	0	0
16	1	1	1	1	1	0	0	0	0	0
17	1	1	1	1	1	0	0	0	0	0
18	1	1	1	1	1	0	0	0	0	0
19	1	1	1	1	1	0	0	0	0	0
20	1	1	1	1	1	1	1	1	0	0
21	1	1	1	1	1	1	1	1	0	0
22	1	1	0	0	1	1	1	0	0	0
23	1	1	0	0	0	1	0	0	0	0
24	1	1	0	0	0	0	0	0	0	0

Table 6. CPU time comparison

METHOD	CPU time					
	No of units					
	10	20	40	60	80	100
GA [8]	221	733	2697	5840	10036	15733
EP [19]	100	340	1176	2267	3584	6120
LRGA [20]	518	1147	2165	2414	3383	4045
DPLR [21]	108	299	1200	3199	8447	12437
GAUC [21]	85	225	614	1085	1975	3547
Proposed LR	10	14	25	39	64	80

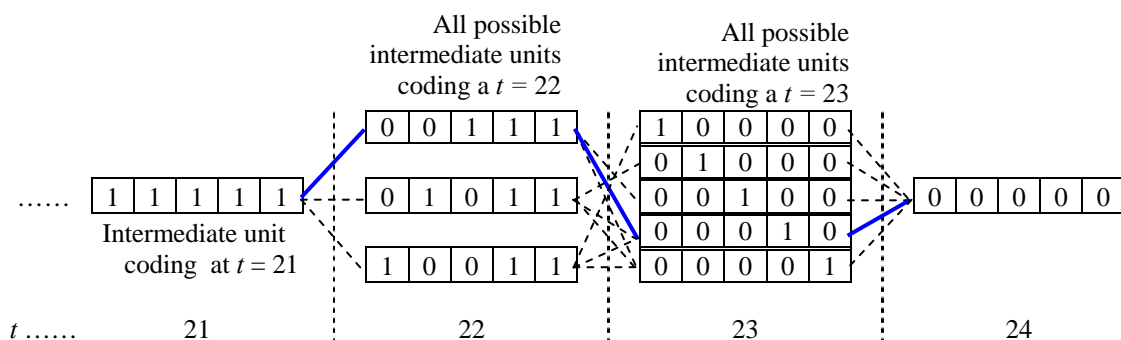


Fig. 1. Dynamic programming search for uncertain stages 22 and 23.