Nonlinear Self-Scheduling of a Single Unit Small Hydro Plant in the Day-Ahead Electricity Market

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Abstract. This paper presents a model to solve the self-scheduling problem of a single unit small hydro plant, based on nonlinear programming techniques. The objective is to maximize the profit obtained by selling energy in the electricity market. The proposed model considers hydro power generation as a function of the water discharge and the net head and takes into account the unit start-ups and shut-downs. The scheduling problem is solved with commercially available optimization software. Input data are introduced through a worksheet, where results are exported once the optimization process is finished. Finally, results from a practical case study are further discussed.

Key words. Competitive electricity market, small hydro plant, self-scheduling, nonlinear programming, head variation, start-ups and shut-downs.

Nomenclature

The notation used throughout the paper is presented next:

k	Index of time periods (stages)
K	Set of indices of time periods
π_k	Forecasted market clearing prices [cent €/kWh]
p_k	Power output in period k [kWh]
Δt	Time periods duration [1 hour]
γ	Specific weight of water [9.81 kN/m^3]
η	Unit global efficiency [p.u.]
q_k	Water discharge in period k [m^3/s]
h_k	Net water head in period k [m]
v_k	Reservoir volume at the end of period k [hm^3]
M	Feasible region of the problem
j	Index of approximating segments
r_k^j	Slope of approximating segment j in period k (hydro plant generation characteristic) [kW/m^3/s]
q_k^j	Water discharge of approximating segment j in period k [m^3/s]
l_k^j	Maximum water discharge of approximating segment j in period k [m^3/s]
f	Conversion factor [0.0036 hm^3/h/m^3/s]
w_k	Forecasted water inflows into the reservoir in period k [m^3/s]
s_k	Spilled flow in period k [m^3/s]
$q_{\max}$	Maximum water discharge [m^3/s]
$v_{\min}$	Minimum reservoir volume [hm^3]
$v_{\max}$	Maximum reservoir volume [hm^3]
s_{ec}	Ecological flow [m^3/s]
b_j	Slope of approximating segment j (spilled flow) [m^3/s/hm^3]
d_j	Maximum volume of approximating segment j (spilled flow) [hm^3]
v_0	Initial reservoir volume [hm^3]
$c_1,\ldots,c_5$	Regression coefficients modeling the hydro plant generation characteristic surface
rsd	Relative standard deviation of the residues [pu]
m_1^j, m_2^j, m_3^j	Polynomial coefficients modeling the slopes of approximating segment j (hydro plant generation characteristic)

1. Introduction

Spanish day-ahead electricity market is based on a pool, as many other electricity markets all over the world, such as the English, Swedish or Dutch ones: generating companies submit bids for selling energy whereas consumers and distributing companies submit bids for buying energy. The market clearing procedure is performed by a private company (the Market Operator) that provides the hourly market clearing prices.

In general, the objective of any power generating company is to maximize the profit obtained from selling energy. Power plants whose influence in market clearing prices is negligible are referred to as price-taker. This paper is aimed at getting the optimal hourly schedule that maximizes the profit a price-taker hydro plant obtains by selling energy in the day-ahead electricity market. Optimal scheduling of this type of power plants is carried out assuming the market clearing prices are known. Forecasting of market clearing prices is therefore outside the scope of this paper. In this respect, several procedures, such as auto regressive models [1] or neural networks [2], have been already presented.

The shorter the term planning is, the more important it is the accurate modeling of the hydro plant generation characteristic [3], that describes the nonlinear and nonconcave relationship existing among the power generated, the water discharge and the net head, and is usually represented as a family of unit performance curves corresponding to different gross head values. No analytical expressions defining this relationship were found in the literature. However, many approximations and heuristic approaches have been proposed to describe the hydro power production phenomenon, such as least squares regression techniques [4] and intelligent control techniques [5].

Several methods have been used to address the hydro self-scheduling problem proposed here: linear programming [6], mixed integer linear programming [7], nonlinear programming [8], dynamic programming [9], methods based on optimal control theory [10, 11], and decomposition techniques [12], among others. Dynamic programming has probably been the most used method to solve this problem since it can handle the non-convexities and nonlinearities of the hydro plant generation characteristic. Nevertheless, it suffers from the so-called curse of dimensionality.

In linear programming based models, the hydro generation characteristic is usually considered as a piecewise linear approximation of a single unit performance curve, thus neglecting the effect of the head variation. A similar limitation was found in mixed integer programming based models, until the appearance of [13], where the generation characteristic was reduced to three unit performance curves, each of which was approximated by a nonconcave piecewise linearization by means of binary variables. Unfortunately, the computational burden required to solve this problem substantially increases with the number of reservoir levels considered.

Polynomial approximations of the generation characteristic are in general more accurate than piecewise linear ones but make the objective function nonconcave, which may result in multiple local optima solutions. In addition, the management of start-ups and shut-downs by means of binary variables leads, in this case, to nonlinear mixed integer problems, generally hard to solve and, in some cases, numerically intractable [14].

An alternative approach, considering both the effect of head variation and start-ups and shut-downs management is presented in this paper. In order to consider the head variation, the hydro generation characteristic is represented as a family of piecewise linear unit performance curves. The first break point of every (piecewise linear) unit performance curve is at the point of zero power, thus allowing to manage the unit start-ups and shut-downs [15].

Due to the short term planning period, the problem is treated as a deterministic one with respect to reservoir water inflows and market clearing prices. Reservoir levels at the end and the beginning of the time horizon considered are assumed known, the final reservoir level being usually specified as a target, obtained from a longer term planning procedure. In all cases analyzed here, the reservoir level at the end of the day was supposed to be identical to its initial value.

This paper is organized as follows. In section II the problem is thoroughly described. A novel approach to approximate the hydro plant generation characteristic surface and an iterative procedure that guarantees the solution feasibility are presented in this section. In section III, results from a practical case study are reported and discussed. Finally, main conclusions of the study are presented in section IV.

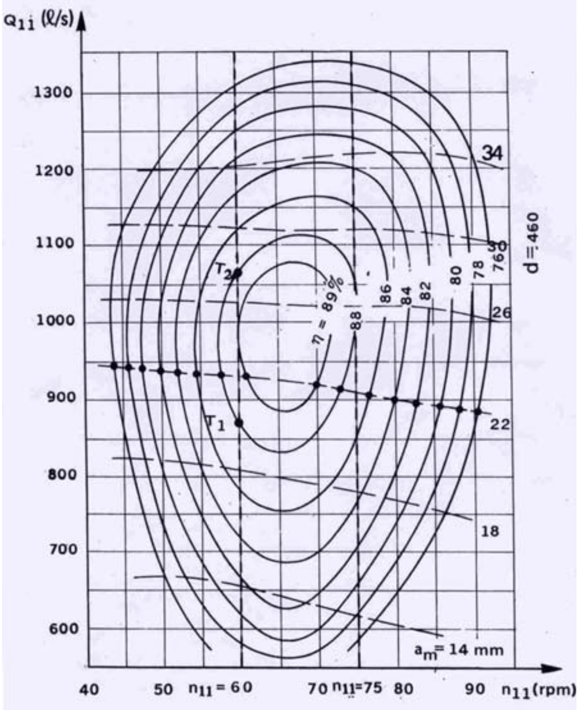


Fig. 1: Turbine hill curves.

2. Formulation

A. Objective function

In general, any power generating company taking part in the day-ahead electricity market aims for maximizing the profit obtained from selling energy. Production costs of a hydro plant can be assumed negligible. Thus, the objective function to be maximized can be expressed as follows:

$$\sum_{k \in K} \pi_k \, p_k \Delta t \tag{1}$$

The optimal plant schedule should maximize (1) while satisfying all constraints described hereinafter.

B. Hydro plant generation characteristic

The power generated by a hydro generating unit depends on the unit global efficiency, the net head and the water discharge, as it is stated in (2).

$$p_k = \gamma \eta_k q_k h_k \tag{2}$$

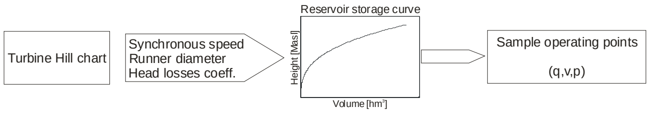


Fig. 2: Sample operating points.

The generator efficiency is considered constant over a wide range of operation whereas the turbine efficiency is described by a nonlinear function of both the water discharge and the net head, embodied in the so-called hill chart (see Fig. 1). The net head depends in turn on both the water discharge through the turbine and the gross head, defined as the height difference between the reservoir water level and the tailrace elevation. It is usual to consider the head losses due to water friction in the penstock and the tailrace level variation as a quadratic function of the turbine water discharge, by means of an appropriate head losses coefficient. Thus, the power generated can be expressed as a function of the reservoir volume and the turbine water discharge, as shown in (3).

$$p_k = f(v_k, q_k) \tag{3}$$

The methodology used to obtain the hydro plant generation characteristic is described below.

1) Least squares fitting

Several operating points defined by the reservoir volume, the water discharge and the power generated, can be obtained from the turbine hill chart and the reservoir storage curve, using an adequate head losses coefficient (see Fig. 2). Then, several mathematical expressions can be tested to fit the hydro plant characteristic surface. In order to avoid multiple local optima solutions, those expressions should be forced to be concave. For a function of two variables, such as (3), to be concave, its Hessian matrix must be negative semidefinite (4). Thus, the obtained operating points can be used as samples to solve the corresponding regression problems by the constrained least squares method.

$$\nabla^2 p(q,v) \le 0\,, \quad \forall (p,q) \in M \tag{4}$$

Once fitted to the sample operating points, it seems clear that (3) is still highly nonlinear. Thus, to consider start-ups and shut-downs by means of binary variables results in a mixed-integer nonlinear problem, that usually requires an in-depth specific analysis before trying any solution procedure.

2) Piecewise linear approximation

The criterion adopted to approximate the generation surface is similar to that proposed in [15]. In that reference, a single unit performance curve is piecewise approximated by two linear segments with three break points at: zero power (shut-down), maximum efficiency and full gate operating points. Authors state that this approximation will cause most stages to be at one of those break points.

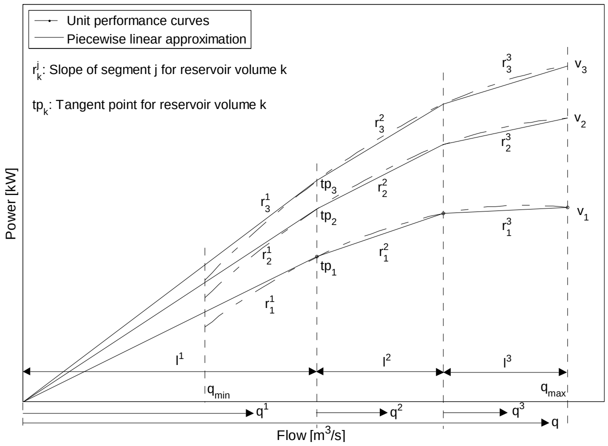


Fig. 3: Family of piecewise linear unit performance curves.

The approach proposed here consists of approximating the hydro generation surface by a family of piecewise linear unit performance curves, in order to consider the head variation. Every (piecewise linear) unit performance curve consists of three segments and four break points located at: the zero power point; the point of tangency with the tangent line from the origin, in such a way as to maintain concavity; the intermediate point that minimizes the area between the unit performance curve and the piecewise linear approximation; and the full gate point (see Fig. 3). The hydro generation characteristic can therefore be expressed as follows:

$$p_k = \sum_{j=1}^{3} r_k^j (v_k) q_k^j \tag{5}$$

$$\sum_{j=1}^{3} q_k^j = q_k \tag{6}$$

$$q_k^j \le l_k^j (v_k) \tag{7}$$

C. Constraints.

The following constraints are to be satisfied for the problem solution to be feasible.

1) Water balance equation

The change in volume of the reservoir must be equal to the water inflow to the reservoir minus the discharge through both the turbine and the spillway.

$$v_{k+1} = v_k + f\left(w_k - q_k - s_k\right) \tag{8}$$

2) Operating limits

Hydro generating units are subject to operating limits so as to prevent the appearance of cavitation, draft tube pressure oscillations, and runaway conditions, and, in some cases, to allow other kind of uses, such as recreational or irrigation uses. Operating limits can be divided into two categories: those expressed in terms of either the power output or the water discharge; and those expressed in terms of the reservoir volume.

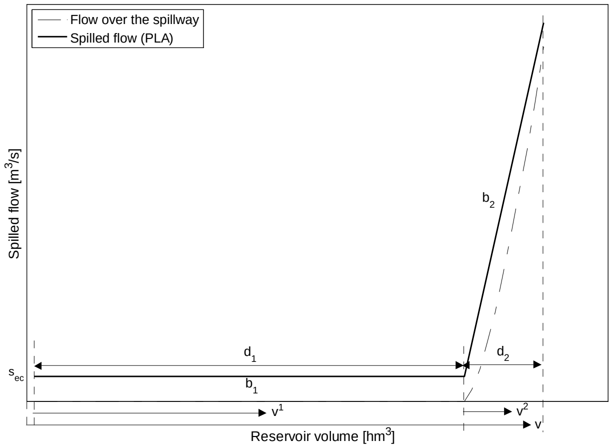


Fig. 4: Spilled flow.

$$q_k \le q_{\max} \tag{9}$$

$$v_{\min} \le v_k \le v_{\max} \tag{10}$$

Minimum flow limit is firstly relaxed to allow the unit start-up and shut-down. It will be considered later, in the iterative procedure (subsection 2.D).

3) *Reservoir spill constraints*

The flow over the spillway is usually given by a polynomial function of the water level above the spillway elevation. As it was done with the generation characteristic surface, it has been piecewise approximated by two linear segments, as it is depicted in Fig. 4.

In recent years, environmental awareness has increasingly constrained hydro plants operation. Minimal flow rates are to be satisfied in the vast majority of rivers in order to maintain the natural habitat for the river basin fauna and flora. In Spain, watershed authorities usually set the so-called ecological flow s_{ec}, which is to be continuously maintained downstream. In a diversion plant this should be:

$$s_k = s_{ec} + \sum_{j=1}^{2} b_j v_k^j \tag{11}$$

$$v_{\min} + \sum_{j=1}^{2} v_k^j = v_k \tag{12}$$

$$v_k^j \le d_j \tag{13}$$

4) *Reservoir target level*

Hydro power reservoirs are usually assigned a target level to be fulfilled at the end of the short-term time period considered. This target level is set based mainly on forecast inflows to the reservoir and unit maintenance schedules, and it is in general calculated by a medium- or long-term planning procedure. This target level can be modeled either with an equality constraint [13] or with a penalty function [7]. In this paper, the reservoir level at the end of the time period is forced to be identical to its initial value.

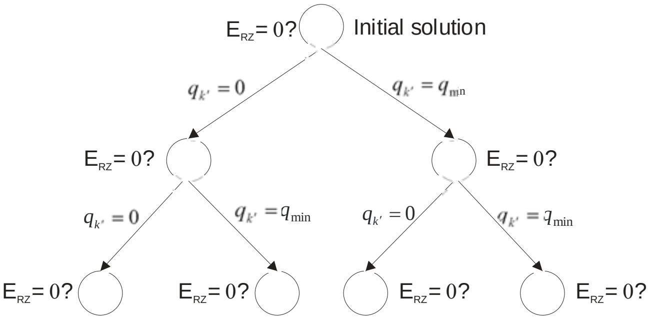


Fig. 5: Iterative procedure.

$$v_{24} = v_0 \tag{14}$$

D. *Iterative procedure*

The problem addressed in this paper consists of maximizing (1) subject to constraints (5)-(14). As in the case reported in [15], the above-proposed approximation (5)-(7) causes most stages to be at one of the four break points of the corresponding piecewise linear performance curve. Then, an iterative procedure is performed in order to solve those cases in which one or more stages fall between the zero-power point and the minimum flow limit.

$$\overline{q} = \left(\overline{q}_1, \overline{q}_2, \ldots, \overline{q}_{24}\right) \tag{15}$$

Let (15) be the initial solution of the problem. Then, stages $k \in K$ may be classified into the following sets:

- E_{SD} Stages such that $\overline{q}_k = 0$
- E_{RZ} Stages such that $\overline{q}_k \in \left(0, q_{\min}\right)$
- E_{OR} Stages such that $\overline{q}_k \in [q_{\min}, q_{\max})$
- E_S Stages such that $\overline{q}_k = q_{\max}$

If $E_{RZ} = \varnothing$, $\overline{q}$ is the optimal solution and iterations are not necessary. On the contrary, if $E_{RZ} \ne \varnothing$, the problem is divided into the following two branches (problems).

- *Branch A.* This branch consists of maximizing (1), subject to (5)-(14) and to the following additional constraint:

$$q_{k'} = 0\,,\; k' \in K \,/\, (\, \pi_{k'} = \min\left(\pi_k\right), k \in E_{RZ}\,) \tag{16}$$

- *Branch B.* This branch consists of maximizing (1), subject to (5)-(14) and to the following additional constraint:

$$q_{k'} = q_{\min}\,,\; k' \in K \,/\, (\, \pi_{k'} = \max\left(\pi_k\right), k \in E_{RZ}\,) \tag{17}$$

Each branch can in turn be divided into two sub branches if the corresponding set E_{RZ} is not an empty set, and so on (see Fig. 5). Additional constraints equivalent to (16) and (17) must be satisfied as iterations progress. When a branch is pruned, the actual hourly power output is calculated with

(3), whereas the actual profit from selling energy is stored. Once iterations stop, the optimal solution is selected among all stored values.

Table I: Generation characteristic surface. Regression coefficients.

c_1	c_2	c_3	c_4	c_5	rsd	$\nabla^2 p(q,v) \leq 0$
-32.54	171.47	564.2	-4.66	-7646	0.0471	Yes

3. Case study

The proposed model has been applied in a practical case study whose main characteristics are described hereafter.

A. Hydro plant generation characteristic surface

Francis turbine hill curves presented in [16] have been used in this study. Table I shows the regression coefficients obtained by least-squares fitting the hydro plant generation characteristic surface as well as the relative standard deviation of the residues. All mathematical expressions tested are based on that proposed in [17], with only small differences, regarding the head losses term, among them. Finally, (18) was selected to fit the surface.

$$p_k(p_k, v_k) = c_1 q_k v_k^2 + c_2 q_k v_k + c_3 q_k + c_4 q_k^2 + c_5 \quad (18)$$

The x-coordinate (q) of the point of tangency of each unit performance curve with the tangent line from the origin turns out to be independent of the reservoir volume. Besides that, the x-coordinate of the third break point varies only slightly with the reservoir volume. Therefore, lengths of the approximating segments are assumed to be constant, their values being given in the last column of Table II. In turn, slopes of the approximating segments prove to be quadratic functions of the reservoir volume (19), their coefficients being also given in Table II.

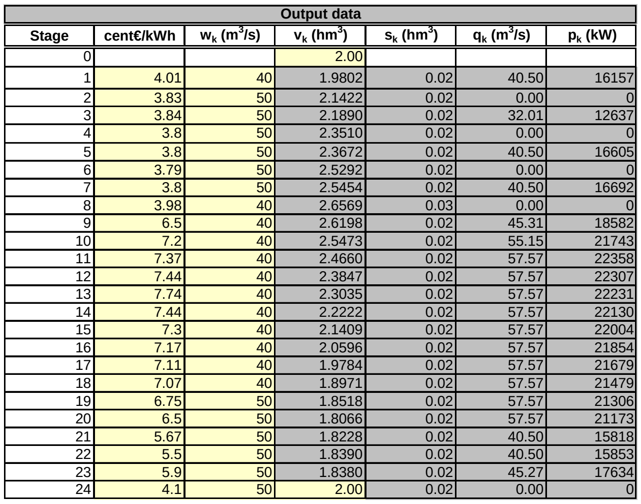
$$r_k^j = m_1^j v_k^2 + m_2^j v_k + m_3^j \quad (19)$$

Table II: Slopes and lengths of approximating segments.

PLA	m_1	m_2	m_3	l
r_1	-32.54	171.47	186.59	40.50
r_2	-32.42	171.20	106.61	17.08
r_3	-32.42	171.20	-54.23	17.43

B. Results

The scheduling problem is solved using the MINOS solver, commercially available for GAMS platform [18]. A user pre- and post-processing algorithm has been implemented in MS Excel in order to collect input and output GAMS data: input data are introduced through a worksheet (see Fig. 6), where results are exported once the optimization process is finished. Both numerical and graphic results are provided (Figs. 6-10) on an hourly basis.



Output data						
Stage	cent€/kWh	w_k (m^3/s)	v_k (hm^3)	s_k (hm^3)	q_k (m^3/s)	p_k (kW)
0			2.00			
1	4.01	40	1.9802	0.02	40.50	16157
2	3.83	50	2.1422	0.02	0.00	0
3	3.84	50	2.1890	0.02	32.01	12637
4	3.8	50	2.3510	0.02	0.00	0
5	3.8	50	2.3672	0.02	40.50	16605
6	3.79	50	2.5292	0.02	0.00	0
7	3.8	50	2.5454	0.02	40.50	16692
8	3.98	40	2.6569	0.03	0.00	0
9	6.5	40	2.6198	0.02	45.31	18582
10	7.2	40	2.5473	0.02	55.15	21743
11	7.37	40	2.4660	0.02	57.57	22358
12	7.44	40	2.3847	0.02	57.57	22307
13	7.74	40	2.3035	0.02	57.57	22231
14	7.44	40	2.2222	0.02	57.57	22130
15	7.3	40	2.1409	0.02	57.57	22004
16	7.17	40	2.0596	0.02	57.57	21854
17	7.11	40	1.9784	0.02	57.57	21679
18	7.07	40	1.8971	0.02	57.57	21479
19	6.75	50	1.8518	0.02	57.57	21306
20	6.5	50	1.8066	0.02	57.57	21173
21	5.67	50	1.8228	0.02	40.50	15818
22	5.5	50	1.8390	0.02	40.50	15853
23	5.9	50	1.8380	0.02	45.27	17634
24	4.1	50	2.00	0.02	0.00	0

Figure 6: MS Excel Worksheet where input and output data are collected.

The performance of the proposed model has been evaluated under different operating conditions, given by different values of: the initial reservoir level and the forecasted water inflows. The hourly market clearing prices considered were taken from the Spanish Market Operator web page [19] on June 28, 2006. The reservoir level at the end of the day was forced to be identical to its initial value. In all cases analyzed, the optimal solution was achieved in less than one minute. The model has been implemented in a Dell PWS 390, Intel dual-core 2.40-GHz processor with 2.00 GB of RAM.

Figs. 7-10 show the evolution of the following variables: reservoir volume, spilled flow, flow through the turbine and power output, in one of the cases analyzed. As it can be seen in Fig. 7, the reservoir stores water during stages preceding peak hours so as to take maximum advantage of the water head in the latter. From peak hours on, the reservoir volume starts decreasing in order to meet the reservoir target level at the end of the time period considered. It is worthwhile to mention that in none of the stages, the operating point lies within the third approximating segment, which is the one with the lowest slope, or energy coefficient ($MW/m^3/s$). It should also be noted that, as it was stated in [15], in most stages, the operating point falls at one of the four corresponding break points. Thus, the greater the number of approximating segments, the more accurate the solution will be. As a matter of fact, the generalization to any number of approximating segments (nonlinear zone) is straightforward.

Results of the proposed model have been compared to those of two more simplified models. In these models a unique performance curve is piecewise approximated, thus neglecting the head variation. The difference between them lies in how the start-ups and shut-downs are tackled: by means of binary variables; or by means of a linear segment, in a similar fashion to that proposed in this paper. In all cases analyzed, both the energy generated and the profit obtained by selling that energy proved to be higher in the proposed model than in either of the others. In addition, the existing mismatch between the value of the objective function and the actual value of the profit is rather higher in the simplified models.

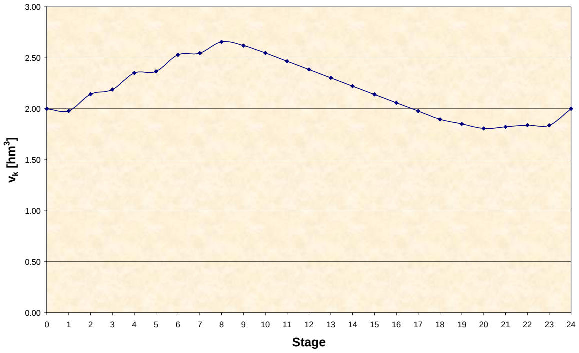


Figure 7: Results. Reservoir volume.

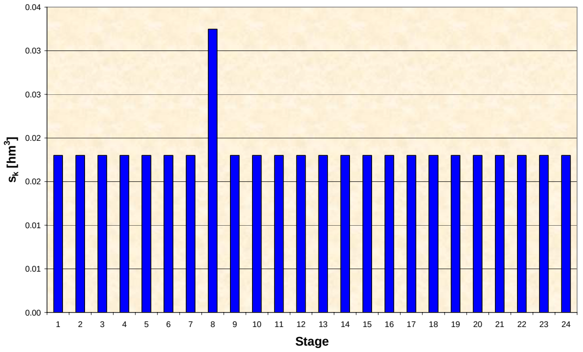


Figure 8: Results. Hourly spilled flow.

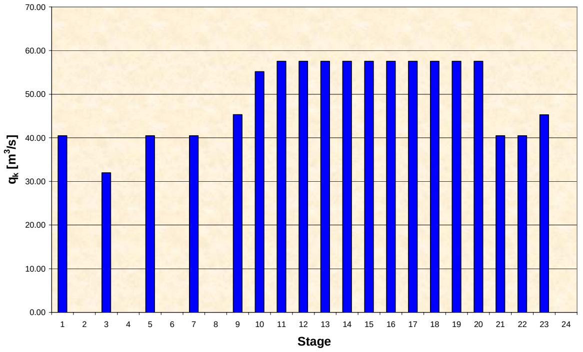


Figure 9: Results. Hourly water discharge through the turbine.

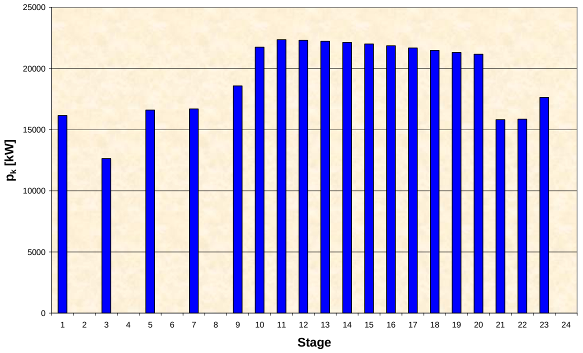


Figure 10: Results. Hourly power output.

4. Conclusions

This paper presents a nonlinear programming model to solve the short-term self-scheduling problem of a single unit small hydro plant. The main contribution of this model is to consider both the head dependence of the unit generation characteristic surface and the unit start-ups and shut-downs.

After deregulation was introduced in electricity markets, generating companies are more interested in the efficient management of energy resources. In this context, the model described here provides hydro power generating companies with a useful tool to make decisions in a short-term (24 h) planning procedure. In order to facilitate the use of this model to a wider class of users, a pre- and post-processing algorithm to collect input and output data has been implemented in an easy-to-use and well-known software program (MS Excel).

The model has been successfully tested on a practical case study under different operating conditions. In addition, results have been compared to those obtained with two other models that neglect the head variation, based respectively on linear and mixed-integer linear programming. In all cases analyzed, the nonlinear model provided higher values of both the energy and the profit, thus demonstrating the importance of considering the head dependence of the unit generation characteristic.

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