

# Ferroresonance in Voltage Transformers: Analysis and Simulations

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## Abstract

Power quality and power disturbances have become an important increasing factor throughout electrical networks. Ferroresonance is one of these disturbances that can occur on distribution systems, causing quality and security problems. This paper analyzes the ferroresonance as a nonlinear resonance phenomenon of erratic nature and difficult prediction. The theoretical principles of this phenomenon and the particular symptoms that identify it will be thoroughly studied. Besides, due to its nonlinear behaviour, several ferroresonance analysis methods commonly used are presented. The theoretical principles are supported by several simulations based on the overvoltage behaviour of voltage transformers.

**Keywords:** Ferroresonance, nonlinear resonance, voltage transformer, overvoltage, transient analysis.

## 1. Introduction

The first published work relative to ferroresonance dates from 1907 and it analyzed transformer resonances [1]. The word ferroresonance was firstly used by Boucherot in 1920 to describe a complex resonance oscillation in a series RLC circuit with nonlinear inductance [2]. Nowadays, ferroresonance is a widely studied phenomenon in power systems involving capacitors, saturable inductors and low losses.

That is why it is a frequent phenomenon in electrical distribution systems, due to the transformers saturable inductance and the capacitive effect of the distribution lines. This capacitive effect is provided by several elements, such as protective elements (circuit breaker grading capacitance), power transmission elements (conductor to earth capacitance, cables capacitance, busbar capacitance, coupling between double circuit lines, capacitor banks), isolation elements (bushing capacitance) or measurement elements (capacitive voltage transformers).

Furthermore, low resistance systems (low-loss transformers, unloaded transformers, low circuit losses) increase the risk of ferroresonant conditions. A good

example of this would be voltage transformers, which are very lightly loaded, as it feeds voltage measuring devices, becoming prone to ferroresonance condition.

The ferroresonance phenomenon appears after transient disturbances (transient overvoltage, lightning overvoltage or temporary fault) or switching operations (transformer energizing or fault clearing). Its effects are characterized by high sustained overvoltages and overcurrents with maintained levels of current and voltage waveform distortion, producing extremely dangerous consequences.

Nevertheless, the ferroresonance phenomenon depends on many other factors and conditions such as initial conditions of the system, transformer iron core saturation characteristic, residual fluxes in the transformer core, type of transformer winding connection, capacitance of the circuit, point-of-wave switching operation or total losses. So its predictability may be considered as quite complex and difficult.

In this paper a software model has been developed to simulate the ferroresonance phenomenon in voltage transformers. It will be used to develop ferroresonance detection methodologies in voltage transformers in a very near future.

## 2. Theoretical Principles of Ferroresonance

Ferroresonance implies resonance with saturable inductance, so it is important to analyze the behaviour of a resonant circuit.

### A. Resonance

In a resonant circuit, inductive and capacitive reactances of the circuit are equal to each other (1). The only opposition to current is the circuit resistance, resulting in undesired overvoltages and overcurrents at the resonance frequency (2). This resonance effect presents one stable operation state, and its effects are mitigated by the system frequencies control or by the introduction of pure resistances.

$$\omega L = \frac{1}{\omega C} \quad (1)$$

$$f = \frac{1}{2\pi\sqrt{LC}} \quad (2)$$

### B. Ferroresonance

Ferroresonance is a resonance situation with nonlinear inductance, so the inductive reactance not only depends on frequency, but also on the magnetic flux density of an iron core coil (e.g. transformer iron core).

The inductive reactance is represented by the saturation curve of a magnetic iron core. Theoretically, this nonlinear inductance could be represented by two inductive reactances, according to the situation on the saturation curve.

- Linear zone  $\Rightarrow X_{L-linear} = \omega L_{linear}$
- Saturation zone  $\Rightarrow X_{L-sat} = \omega L_{sat}$

Like resonance, depending on the connection between the capacitance and the nonlinear inductance, ferroresonance may be series or parallel. This paper analyzes only series ferroresonance (3) (Fig. 1).

$$\vec{E} = \vec{V}_L + \vec{V}_C \quad (3)$$

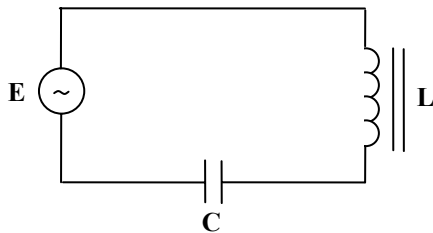


Fig. 1. Series ferroresonant circuit

Figure 2 (a) shows the graphical solution of a series ferroresonant circuit. The possible operation points are obtained as the intersection of the lines  $V_L$  and  $E+V_C$ , where  $E$  is the voltage source,  $V_C$  the capacitance voltage and  $V_L$  the inductance voltage (Fig. 2a). Another way to represent this graphical solution consists of the previous calculation of the line  $E$ , as it is shown in figure 2 (b) [3]. Both representations show three possible operation points.

- Operation point 1: It is a non-ferroresonant stable operation point. This is an inductive situation ( $X_{L-linear} > X_C \Rightarrow E = V_L - V_C$ ).
- Operation point 2: It is a ferroresonant stable operation point. This is a capacitive situation ( $X_{L-sat} < X_C \Rightarrow E = V_C - V_L$ ).
- Operation point 3: It is an unstable operating point.

As it can be seen in these figures, the main characteristic of a ferroresonant circuit is to present two stable operating states at least, producing sudden leaps of

voltage or current from one stable operating state to another one.

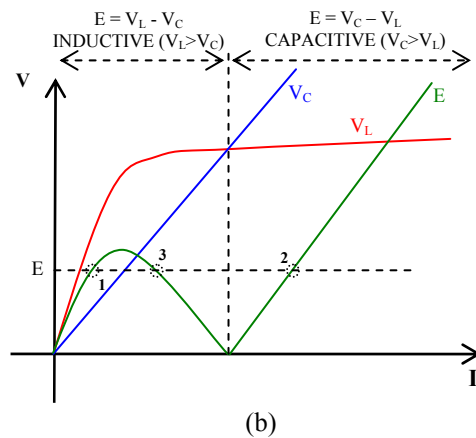
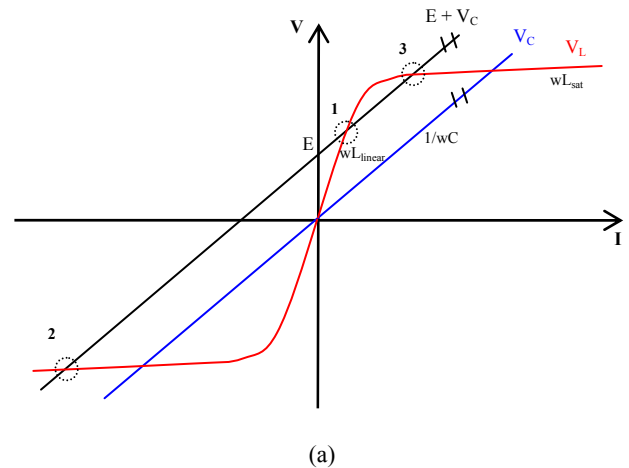


Fig. 2. Graphical solution of a series ferroresonant circuit

It is possible to verify that the step from a non-ferroresonant operation point to a ferroresonant one coincides with the step from an inductive situation to a capacitive one (Fig. 2b). The final operation point will depend on initial conditions (residual flux, capacitance value, voltage source, switching instant,...). This way, under certain initial conditions (e.g. transient overvoltage), ferroresonance can appear. This ferroresonant point gets to overvoltage and overcurrent situations. Once the ferroresonance has appeared, the system stays working under ferroresonance, until the source is not able to provide the necessary energy to maintain the phenomenon.

### C. Evolution of ferroresonant operation point

This section analyzes the evolution of the solution according to the value of the  $E+V_C$  line.

Figure 3 shows this evolution while the voltage source value is changed, maintaining constant the capacitance value. Similarly, figure 4 shows the evolution when the capacitance value changes and the voltage source value maintains constant.

In the first case (Fig. 3), the straight line  $E+V_C$  moves parallel to itself. There is a voltage source value ( $E$ ) for which the straight line  $E+V_C$  is tangent to the magnetization curve. With higher voltage source values ( $E''$ ), the system solution is always a ferroresonant situation. On the contrary, with lower voltage source values ( $E'$ ), the system has two possible stable operation points: one ferroresonant point and another non-ferroresonant point. The final operation point will depend on the initial conditions.

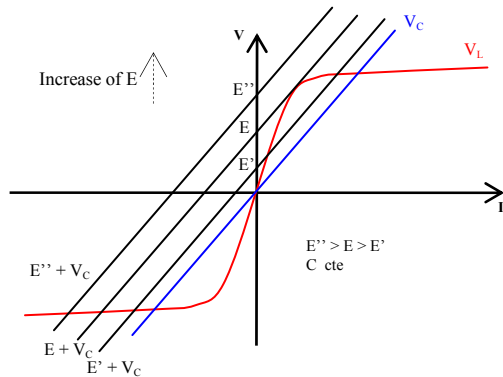


Fig. 3. Evolution of the solution increasing source voltage  $E$

In the second case (Fig. 4), the straight line  $E+V_C$  rotates around the point  $E$ , so its slope decreases as the capacitance is increased. As it can be seen in figure 4, unlike resonance state, ferroresonant state is possible to appear for a wide range of capacitance values at a given frequency [4].

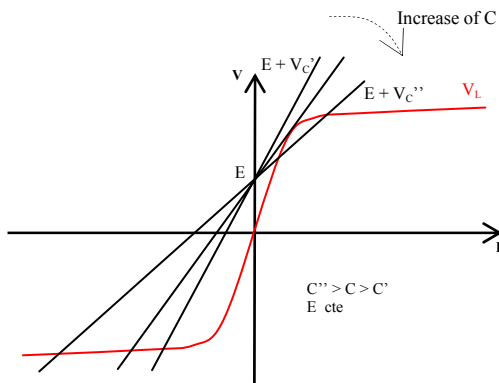


Fig. 4. Evolution of the solution increasing capacitance value  $C$

Thus, the theoretical conditions to avoid periodic ferroresonance depend on the inductance values of the magnetization curve, as it can be observed in (4). This condition is represented in figure 5.

$$\omega \cdot L_{\text{linear}} < \frac{1}{\omega \cdot C} < \omega \cdot L_{\text{sat}} \quad (4)$$

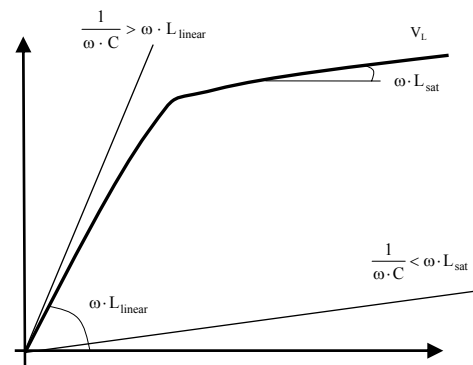


Fig. 5. Periodic ferroresonance conditions

#### D. Phenomena relative to ferroresonance

Although ferroresonance is a phenomenon with high prediction difficulties, some phenomena related to ferroresonance have been provided through the years. These phenomena can help to identify a ferroresonant situation. Some of them are [5]-[8]:

- Overvoltages and overcurrents
- Sustained levels of distortion
- Loud noise (magnetostriction)
- Misoperation of protective devices
- Overheating
- Electrical equipment damage
- Insulation breakdown
- Flicker

### 3. Analysis of ferroresonance

Depending on circuit conditions, different ferroresonant states can be obtained. Before proceeding to study these ferroresonant states, some commonly used analysis methods of nonlinear dynamic systems are described below.

#### A. Analysis methods

There are several tools available to study nonlinear dynamic systems [9]-[12]. Most of them are based on bifurcation theory. This theory studies the evolution of the system solution changing the value of a control parameter. However, in this paper the ferroresonant behaviour is analyzed based on three different methods: spectral density, phase plane and Poincaré map.

- 1) *Spectral density*: The harmonic component of the voltage and current signals is analyzed with this method. The FFT analysis is used to obtain the characteristic frequencies present in the signals. The presence of more than one characteristic frequency shows the multiplicity in periodicity, commonly presented in some ferroresonant states.

- 2) *Phase plane*: The time behaviour of a system is analyzed with this graphical method. It is just a plot of two state variables of the system (e.g. voltage and flux). The result is the temporal evolution of a point following a trajectory. Periodic solutions correspond to closed trajectories.
- 3) *Poincaré map*: This method is similar to phase plane method, but in this case, the sampling frequency is the same as the system frequency, so the Poincaré map of a periodic solution consists of a single point.

#### B. Ferroresonant modes

According to the steady state condition, ferroresonant states can be classified into four different types:

- 1) *Fundamental mode*. The signals (voltage and current) have a distorted waveform, but are periodic with a period equal to the source period (T). The signal spectrum is made up of the fundamental frequency of the system (f) and its harmonics. The phase plane shows a single closed trajectory and the Poincaré map shows one point moved far away from the normal state point.
- 2) *Subharmonic mode*. The signals are periodic, but the period is an integer multiple of the source period (nT). These states are known as subharmonics of order 1/n. The signal spectrum is made up of a fundamental component (f/n) and its harmonics. The phase plane shows a closed trajectory with n sizes and the Poincaré map shows n points.
- 3) *Quasi-periodic mode*. In this mode, the signals are not periodic. The signal spectrum is still a discontinuous spectrum. The phase plane shows changing trajectories and the Poincaré map shows several points representing a closed curve.
- 4) *Chaotic mode*. The signals show an irregular and unpredictable behaviour. The signal spectrum is continuous. The phase plane shows trajectories that never close on themselves and the Poincaré map shows several points with not fixed pattern.

### 4. Ferroresonance Simulations

The ferroresonance phenomenon in voltage transformers is analyzed in this section through several software simulation examples, using the software tool MATLAB. A software model has been developed based on test data obtained on a real voltage transformer. Figure 6 shows the saturation curve of the voltage transformer model developed.

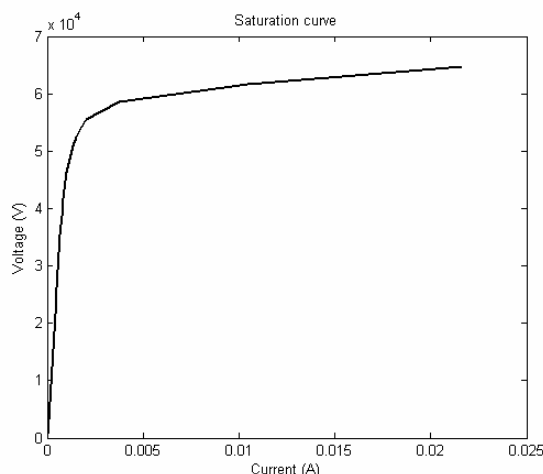


Fig. 6 Saturation curve of the voltage transformer model

This software model has been used to simulate the energization of a no-load voltage transformer, analyzing the behaviour of the model with the variation of initial conditions, such as voltage source amplitude, capacitance value or switching instant (residual flux has not been considered). The critical capacitance values for the periodic ferroresonance appearance have been calculated based on the model saturation curve and equation (4). According to the capacitance values obtained, periodic ferroresonance is possible to appear with capacitance values between 0.06 nF and 0.01  $\mu$ F approximately. Thus, the parameters values to run the simulations have been determined, such as it is shown in table I.

TABLE I. – Parameters of the Simulations

Parameter	Values	
	From...	To...
Capacitance	1e-5 $\mu$ F	0.01 $\mu$ F
Voltage Source (RMS)	15 kV	50 kV
Degrees (switching instant)	0°	180°

In order to analyze the behaviour of the voltage transformer model with the variation of switching instant and capacitance value, several simulations at 25 kV voltage source are shown below.

#### A. Switching instant

The switching instant is an important factor in the ferroresonance analysis. Its influence is similar to the inrush currents effect in the transformer energization. In this section three simulations with three different switching instant are shown. The simulation parameters are presented in Table II.

TABLE II. – Parameters – Switching Instant

Parameter	Value	
Capacitance	4e-4 $\mu$ F	
Voltage Source (RMS)	25 kV	
Degrees (switching instant)	Simulation A1	90°
	Simulation A2	30°
	Simulation A3	0°

The graphical solution of these simulations is shown in figure 7. Simulation A1 is a non-ferroresonant situation. Simulations A2 and A3 are ferroresonant situations.

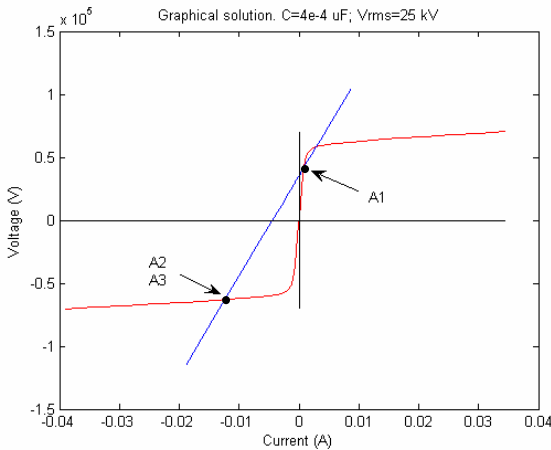


Fig. 7. Graphical solution energizing at 25 kV and  $4e-4 \mu\text{F}$

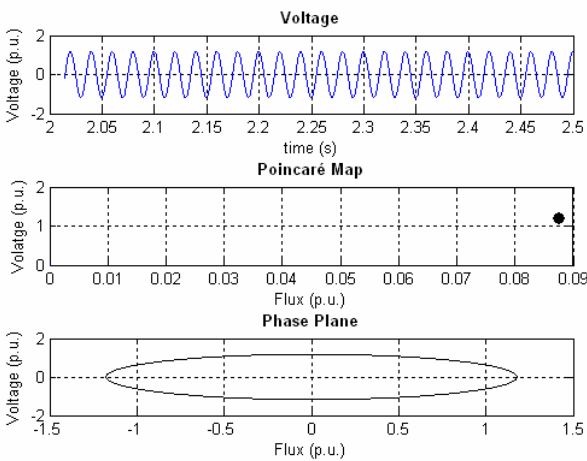


Fig. 8. Simulation A1. Voltage waveform, Poincaré map and phase plane energizing at 25 kV, 90 degrees and  $4e-4 \mu\text{F}$ . Non-ferroresonant situation.

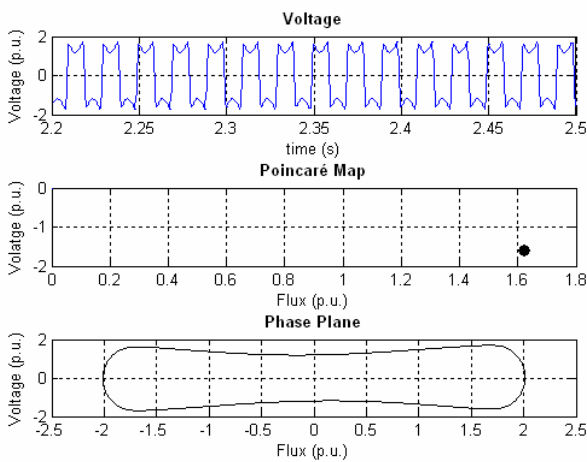


Fig. 9. Simulation A2. Voltage waveform, Poincaré map and phase plane energizing at 25 kV, 30 degrees and  $4e-4 \mu\text{F}$ . Fundamental ferroresonance.

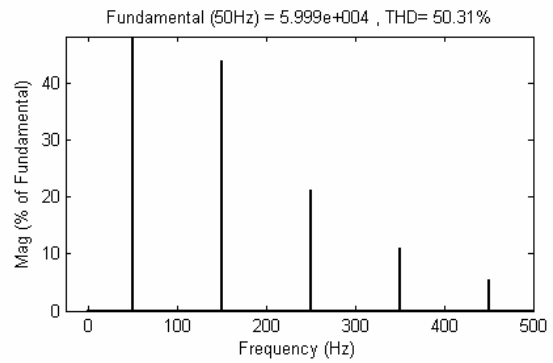


Fig. 10. Simulation A2. Voltage spectral density energizing at 25 kV, 30 degrees and  $4e-4 \mu\text{F}$ . Fundamental ferroresonance

Energizing at 90 degrees, ferroresonant state does not appear, as it can be seen in figure 8. On the contrary, energizing at 30 degrees, fundamental ferroresonance has been obtained, with an overvoltage of 1.7 p.u. The Poincaré map shows only one point and the phase plane shows a closed curve (Fig. 9). The voltage spectral density plot shows the fundamental frequency component at 50 Hz and its harmonics of odd order (Fig. 10).

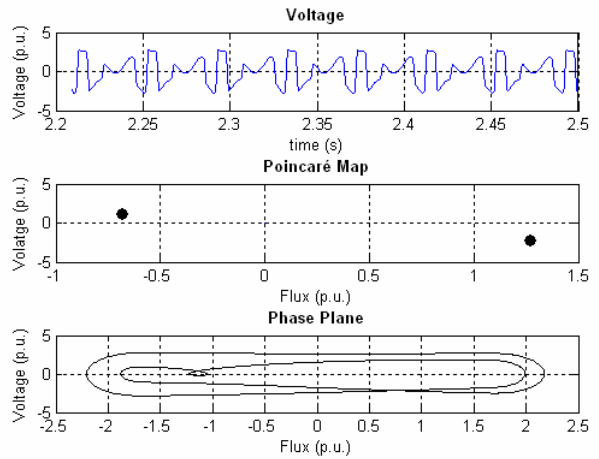


Fig. 11. Simulation A3. Voltage waveform, Poincaré map and phase plane energizing at 25 kV, 0 degrees and  $4e-4 \mu\text{F}$ . Subharmonic ferroresonance.

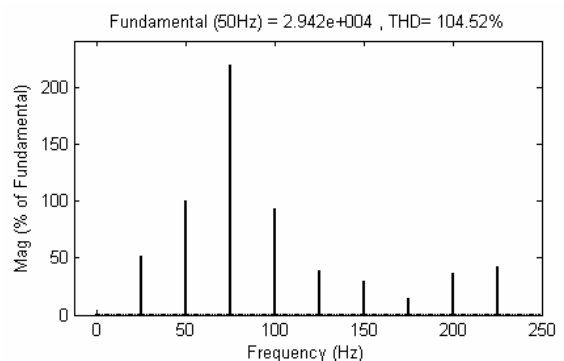


Fig. 12. Simulation A3. Voltage spectral density energizing at 25 kV, 0 degrees and  $4e-4 \mu\text{F}$ . Subharmonic ferroresonance.

On the other hand, when energizing at 0 degrees subharmonic ferroresonance of order  $\frac{1}{2}$  has been obtained with an overvoltage of 2.7 p.u. The Poincaré map shows two points and the phase plane shows one closed trajectory with two sizes (Fig. 11). The voltage

spectral density plot evidences the presence of the 25 Hz (i.e.  $\frac{1}{2}$ ) subharmonic oscillation (Fig. 12).

These three simulations show clearly the influence of the switching instant. The system is more prone to ferroresonance when energizing near to 0 degrees.

### B. Capacitance variation

In this section the evolution of the ferroresonant states with the capacitance value is analyzed. The energization is fixed at 0 degrees. The simulation parameters used are shown in table III.

TABLE III. – Parameters – Capacitance Variation

Parameter	Value	
Voltage Source (RMS)	25 kV	
Degrees (switching instant)	0°	
Capacitance value	Simulation B1	2e-4 $\mu$ F
	Simulation B2	4e-4 $\mu$ F
	Simulation B3	0.01 $\mu$ F

The graphical solution of these simulations is shown in figure 13. The three simulations are ferroresonance situations. The graphical solution of simulation B1 only shows one point, and this point is a ferroresonant point, not depending on switching instant, with an overvoltage of 1.3 p.u. (Fig. 14-15).

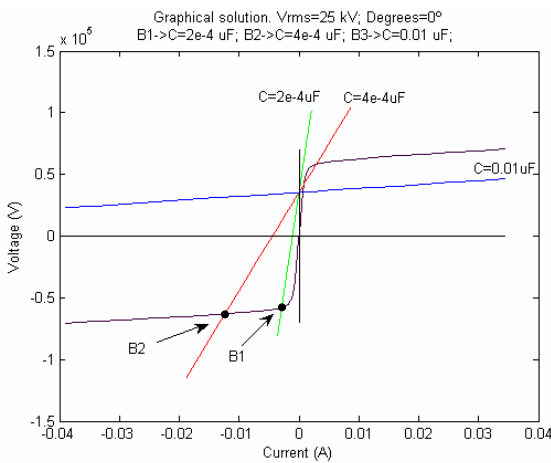


Fig. 13. Graphical solution energizing at 25 kV and 0 degrees, changing the capacitance value.

Simulation B2 is the same as simulation A3 of the previous section. In this simulation, the capacitance value has been increased slightly to 4e-4  $\mu$ F, so the slope of the line  $E+V_C$  has decreased. Consequently, the graphical solution shows two possible operation points (ferroresonant point and non-ferroresonant point). In this case, the final solution is a ferroresonant state (subharmonic ferroresonance of order  $\frac{1}{2}$ ), as it has been shown in section A (Fig. 11-12)

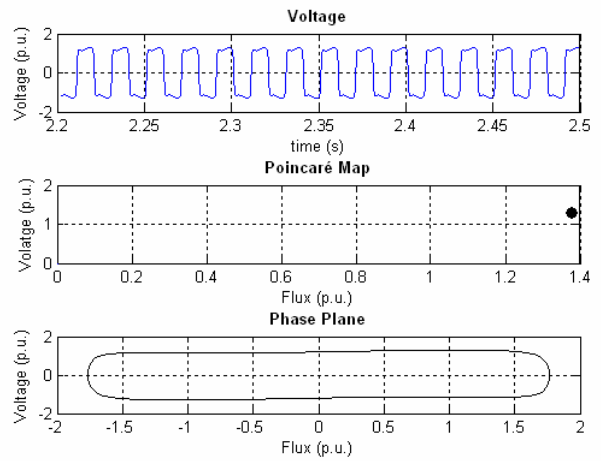


Fig. 14. Simulation B1. Voltage waveform, Poincaré map and phase plane energizing at 25 kV, 0 degrees and 2e-4  $\mu$ F. Fundamental ferroresonance.

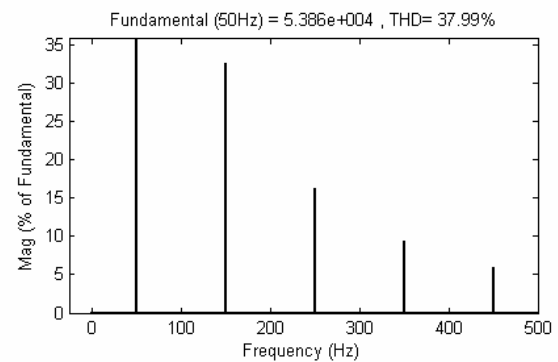


Fig. 15. Simulation B1. Voltage spectral density energizing at 25 kV, 0 degrees and 2e-4  $\mu$ F. Fundamental ferroresonance.

Finally, in the third simulation (B3), the capacitance value has been increased to the limit value of 0.01  $\mu$ F. Under these conditions, a chaotic ferroresonance with an overvoltage of 3.8 p.u. has been obtained. The Poincaré map shows a random set of points without any fixed pattern and the phase plane shows a trajectory that never closes on itself (Fig. 16). The voltage spectral density plot shows a continuous spectrum, typical of a chaotic situation (Fig. 17).

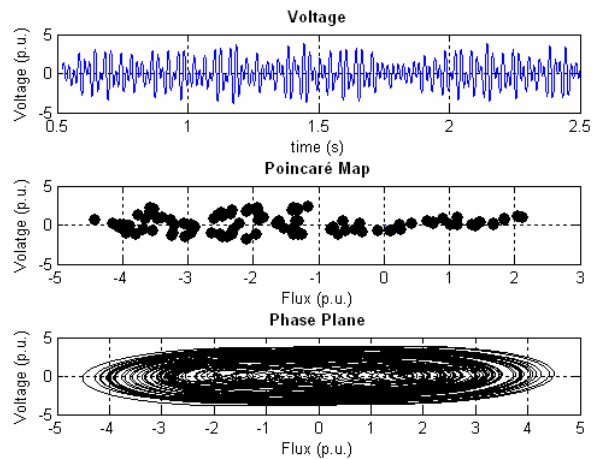


Fig. 16. Simulation B3. Voltage waveform, Poincaré map and phase plane energizing at 25 kV, 0 degrees and 0.01  $\mu$ F. Chaotic ferroresonance.

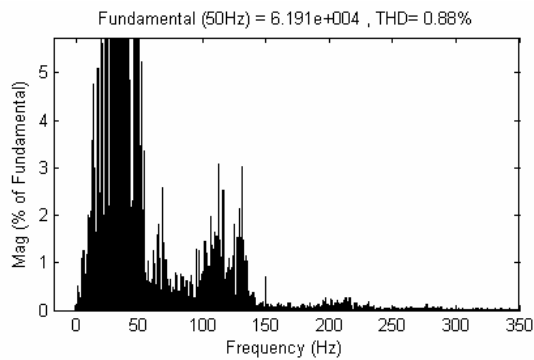


Fig. 17. Simulation B3. Voltage spectral density energizing at 25 kV, 0 degrees and 0.01  $\mu$ F. Chaotic ferroresonance.

## 5. Conclusions

Ferroresonance is a widely studied phenomenon but it is still not well understood because of its complex behaviour. Its effects on electrical equipments are still considerable nowadays. An extended analysis on the ferroresonance phenomenon, its theoretical principles, causes and effects are presented in this paper.

In addition, a software model has been developed to simulate ferroresonant situations, and different ferroresonant modes have been obtained with satisfactory results. The transformer energization has been presented as a critical situation for the ferroresonance to appear. The influence of switching instant and capacitance value has been analyzed through several software simulations, considering the critical capacitance values. Related to the influence of switching instant, it has been demonstrated that the system is prone to ferroresonance with switching instants near to zero degrees. Anyway, ferroresonance depends on many other factors and parameters as it has been mentioned previously in the paper.

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## References

- [1] Bethenod, J., "Sur le Transformateur à Résonance", *L'Éclairage Électrique*, vol. 53, Nov. 30, 1907, pp. 289-96.
- [2] Boucherot, P., "Existence de Deux Régimes en Ferroresonance", *Rev. Gen. de L'Élec.*, vol. 8, no. 24, December 11, 1920, pp. 827-828.
- [3] Berrostequieta, J. "Introduction to instrument transformers" *Electrotécnica Artech Hnos., S.A*
- [4] Ferracci, P., "Ferroresonance", *Groupe Schneider: Cahier technique no 190*, March 1998.
- [5] Santoso, S., Dugan, R. C., Grebe, T. E., Nedwick, P. "Modeling ferroresonance phenomena in an underground distribution system" *IEEE IPST '01 Rio de Janeiro*, Brazil, June 2001, paper 34
- [6] Slow Transient Task Force of the IEEE Working Group on Modeling and Analysis of System Transients Using Digital Programs, "Modeling and analysis guidelines for slow transients – Part III: The study of ferroresonance," *IEEE Trans. on Power Delivery*, vol. 15, No. 1., Jan. 2000, pp. 255 – 265.
- [7] Jacobson, D. A. N. Member, IEEE "Examples of Ferroresonance in a High Voltage Power System" *IEEE Power Engineering Society General Meeting*, 2003:1206-1212.
- [8] Tanggawelu, B., Mukerjee, R.N., Ariffin, A.E. "Ferroresonance studies in Malaysian utility's distribution network" *IEEE Power Engineering Society General Meeting*, July 2003, pg 1219 Vol. 2
- [9] Kieny, C., "Application of the bifurcation theory in studying and understanding the global behavior of a ferroresonant electric power circuit", *IEEE Trans. on Power Delivery*, Vol. 6, No. 2, pp. 866-872, April 1991.
- [10] Preecha S., Somchai Ch. "Application of PSCAD/EMTDC and Chaos Theory to Power System Ferroresonance Analysis" *International Conference on Power Systems Transients IPST05 paper 227* Canada June 2005
- [11] B.A. Mork, D.L. Stuehm, "Application of nonlinear dynamics and chaos to ferroresonance in distribution systems" *IEEE Transactions on Power Delivery* Vol9 No2 April 1994
- [12] A. E. A. Araujo, A. C. Soudack, J. R. Martí, "Ferroresonance in power systems: chaotic behaviour," *IEE Proc.-C*, Vol. 140, No. 3, pp. 237-240, May 1993.