

Estimation of Weibull parameters in winds speed mixture using non-linear optimization for wind energy applications

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Abstract. Climate change and global warming are problems need to be tackled on a priority basis. The greenhouse gas (GHG) emissions and air pollution must be reduced by 25% and 40% compared to 1990 levels in 2020 and a reduction between 80% and 95% by 2050. To mitigate the GHG emissions, countries have adopted policies to use renewable energy sources. In the case of wind energy, the statistical analysis of wind data is a crucial stage for estimating the wind turbine energy output through the turbine performance. The Weibull distribution has been widely used in the recent years for describing the behavior of the wind speed and it can be treated as a probability density function. Herein, it is presented a new method for calculating the Weibull parameters of an infinity sum of Weibull distributions. This new method is based on a Hilbert space generated by scale and form factor as Fredholm integral. This new method is named Inversion of the Weibull Distribution in wind speed mixture (IWeD). The simulations results indicate that IWeD is adequate for estimating the Weibull parameters when the wind speed is composed of several Weibull distributions.

Key words. Weibull distribution, wind speed mixture, Wind energy, Inversion winds speed mixture.

1. Introduction

Global warming and climate change are problems that countries and, in general terms, the society must face [1-5]. The growing demand for fossil fuels and environmental concerns such as greenhouse gas (GHG) emissions and air pollution has raised attention to renewable resources available for attending the growing demand of energy. In 1997, the most industrialized countries pledged, in Kyoto, to execute a set of measures to reduce GHG emissions by at least 5% on average between 2008 and 2012 compared to 1990, entering into legal force in 2005. In this aspect, the Kyoto protocol proposes efficiently consume oil and coal, adopting renewable forms of energy. Industrialized countries should reduce their GHG emissions by 25% and

40% compared to 1990 levels in 2020 and a reduction between 80% and 95% by 2050 [6].

In order to reduce the dependence on fossil fuels for mitigating the GHG emissions, countries have adopted policies to use renewable energy sources. Energy plays an important role in economic development and well-being of the society. Wind energy is widely recognized as one of the most sustainable sources of electricity, with a potential energy generation up to 40 times the annual global electricity consumption [7]. The environmental policies in the European Union (EU) and many other countries favor the growth in the production of wind energy, being them important to study the wind properties and its potential in terms of electricity generation [8].

Wind power is converted through a turbine, where the kinetic energy contained in the airflow turns the rotor of an electric generator. The wind turbine power output (P_t), depends on the probability density function (PDF) of wind speed (v). The wind turbine power output (P_t), therefore, can be expressed by

$$\bar{P}_t = \int_0^{\infty} P_t(v)f(v)dv \quad (1)$$

where $P_t(v)$ is the wind turbine power output as function of the wind speed (v) and $f(v)$ is the PDF.

Wind power in every region must be evaluated considering two main features: the potential and the feasibility [9]. The evaluation of the wind resource, the speed distribution and the average wind power must be studied among other parameters. Normally, several distributions of wind power are mixed and can generate differences in energy for turbines at nearby locations [10]. The statistical methods calculate the probability of the wind speed, and therefore, it is possible to determine the potential of the wind power.

The statistical analysis of wind data is the most crucial step for estimating the PDF as (v) function. This analysis determines the calculation of the available energy and the

wind turbine performance, which have a significant influence on the investment profitability. There are a lot of PDF for describing the behavior of wind data. The commonly used are beta function, lognormal function, Rayleigh and Weibull function [11-12].

The most used probability density function (which describes the wind speed distribution) is the Weibull function [13-14]. This function is widely studied in other engineering areas, as well as medical, chemistry and physics. The wind conditions are such that considering a mixture of Weibull distribution is more suitable for the description in real world. Mixture Weibull distribution is more useful because it can also show heterogeneous wind regimes, in which there is evidence of multimodality [15-16].

There are several numerical methods that can estimate the parameters of the Weibull distribution such as maximum likelihood estimation [17], least squares estimation [18], graphic method, moment method [19], power density method [20], among others.

The Weibull function is defined by the following parameters,

$$F(v) = 1 - e^{-\left(\frac{v}{\lambda}\right)^\kappa} \quad (2)$$

Where $F(v)$ is the distribution function, κ is the shape factor and λ is the scaling factor. The parameters κ and λ are dimensionless parameters. The usual frequency distribution can be assumed and characterized by a shape factor of $\kappa=2$. In this case, this function is called Rayleigh function, which is a special distribution type of Weibull function. On real locations this parameter κ may vary from 1.5 to 2.5. For offshore locations, this parameter is about 1.5. Over land this parameter reaches values up to 2.5 or more depending on the specific location.

The Weibull distribution is defined by a probability density function, which is based on two factors named form factor κ and scale factor λ , since is the derivative respect to wind speed of $F(v)$. This probability density function is given by,

$$f(v) = \frac{\kappa}{\lambda} \left(\frac{v}{\lambda}\right)^{\kappa-1} e^{-\left(\frac{v}{\lambda}\right)^\kappa} \quad (3)$$

Where $f(v)$ is the probability of measuring wind speed v . This measurement of wind speed is performed at one height, but it is also possible to get an estimation at different heights [9].

The wind speed description is considered as a unique Weibull distribution. Sometimes, it is possible to take into account that the wind speed it is actually a sum of Weibull distributions. Equation (2) could be replaced by the following sum equation (3) that describes de probability of the wind speed.

$$g(v) = \sum_{i=1}^n f(v)_i = \sum_{i=1}^n \left(\frac{\kappa_i}{\lambda_i}\right) \left(\frac{v}{\lambda_i}\right)^{\kappa_i-1} e^{-\left(\frac{v}{\lambda_i}\right)^{\kappa_i}} \quad (4)$$

The sum of Weibull distributions can be derived as Fredholm integral. This integral equation has the form as follow,

$$g(r) = \iint_a^b f(t,s)K(t,s,r)dt ds \quad (5)$$

In this integral equation, $g(r)$ and $K(t,s,r)$ are known functions and the integration endpoints 'a' end 'b' are finite or infinite. Normally, K is named kernel of the function, but it is a compact operator form a Banach space [21]. The goal is to solve for $f(t,s)$. For a continuous sum of Weibull distribution for any value of κ and λ , equation (3) could be replaced by the following integral equation (5) through the equation (4). Then, equation 3 is transformed to,

$$g(v) = \iint_0^\infty A(\kappa,\lambda) \frac{\kappa}{\lambda} \left(\frac{v}{\lambda}\right)^{\kappa-1} e^{-\left(\frac{v}{\lambda}\right)^\kappa} d\kappa d\lambda \quad (6)$$

Any value of κ and λ , $A(\kappa,\lambda)$ is a matrix with the lengths of the two functions, which correspond with the parameters κ and λ . The equation (5) could be also transformed into discrete space through the inner product of the $A(\kappa,\lambda)$ and the integral matrix of Weibull function. This discrete space is defined by the Hilbert space, which simplify the mathematical treatment. Equation (4) could be used as a linear algebraic form described by equation (4).

$$g_v = HS \cdot A(\kappa,\lambda) \quad (7)$$

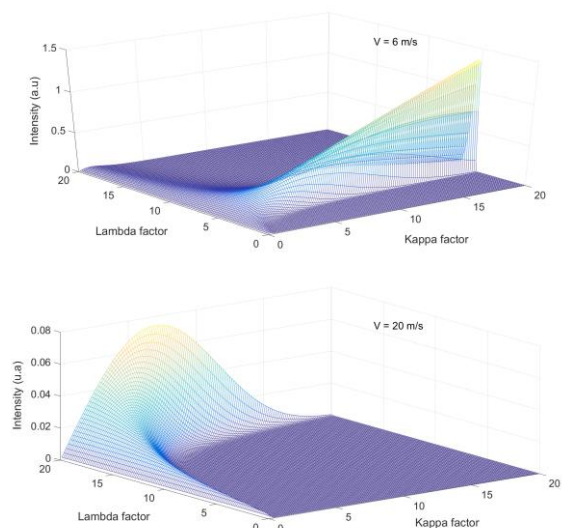


Fig. 1. Aspect of the Hilbert space for two different positions in the dimension m , which is determined by wind speed. Top is the position for wind speed equal to 6 and bottom is the position for wind speed equal to 20.

where g_v is equal to $[g_{v1}, g_{v2}, \dots, g_{vm}]^T$, the term A corresponds to a matrix of $n \times r$ elements of contribution of κ and λ factors, and HS , equation (8), is the $m \times n \times r$ matrix corresponding of the Hilbert space defined by equation (5),

$$HS_{mnr} = \frac{\kappa_n}{\lambda_r} \left(\frac{v_m}{\lambda_r}\right)^{\kappa_n-1} e^{-\left(\frac{v_m}{\lambda_r}\right)^{\kappa_n}} \quad (8)$$

The equation (8) can be visualized in the Figure 1, where they are observed different positions of the dimension m corresponding to wind speed. In this case, wind speed equal to 6 and equal to 20 is shown.

In general terms, the values of $A(\kappa, \lambda)$ can be estimated from experimental data and g_v using the inversion of the equation (6). The discretization of Fredholm integral equations create discrete ill-posed problems. Therefore, this inversion is an ill-posed problem which was originally introduced by Hadamard [22]. Since the initial and boundary conditions are not well-defined, eventually strong vulnerability to noise and numerical instability have induced the emergence of different approaches. In order to face these problems, we can regard Levenberg–Marquardt statistical method [23] and SPLMOD [24]. All these approaches consider solutions with discrete values. Instead, other methods consider the solution as a distribution are CONTIN [25] maximum entropy (MaxEnt) [26] and more recently Proximal Algorithm for L1 combined with MaxEnt prior [27]. This problem can be solved using the widely known Tikhonov regularization method [28] for ill-posed problems and its formulation yield,

$$\min \|HS \cdot A - g_v\| + \alpha f(A) \quad (9)$$

$f(A)$ is the regularization function, and α balances the solution overfitting, such that the value of cost function is penalized on the basis of some prior assumptions. The value of α should be carefully chosen with respect to the noise level. The function of $f(A)$ imposes the solution form. Prior assumptions are included using $f(A)$, this function is represented with different forms,

$$f(A) = \begin{cases} \|LA\|_2^2 \\ \sum \left(\frac{A_i}{\sum A_i}\right) \log\left(\frac{A_i}{\sum A_i}\right) \\ \|A\|_1 \end{cases} \quad (10)$$

In this case, it is proposed the L_1 norm of A as regularization function. This approach endeavors on the recovery of sparse solutions by using the l_1 -norm penalty function. The minimum of the equation (8) can be found by using very robust iterative thresholding algorithms (ISTAs) [29].

This method based on l_1 -norm promotes sparsity of the solutions $A(\kappa, \lambda)$. This implies assumption that the number of components is smaller as possible. Normally, ISTA algorithm itself has a rate of convergence proportional to $1/k$, where k is el number of iterations. Nesterov’s module is applied to the ISTA algorithm, as accelerated method for increasing up to $1/k^2$ the rate of convergence [30-31]. This module was executed as the following sequence. Note that $\gamma_k \leq 0$.

$$\lambda_0 = 0, \lambda_k = \frac{1 + \sqrt{1 + 4\lambda_{k-1}^2}}{2}, \gamma_k = \frac{1 - \lambda_k}{\lambda_{k+1}} \quad (11)$$

$$A'_{k+1} = (1 - \gamma_k)A_{k+1} + \gamma_k A_k \quad (12)$$

This algorithm is called fast iterative shrinkage-thresholding algorithm (FISTA) [32]. This algorithm is derived from ISTAs algorithms.

In this work, we present the application of FISTA to decompose mixture of wind speed distributions.

2. Methods

The FISTA algorithm for decomposing wind speed mixture is evaluated in different simulations. This method is named Inversion of the Weibull Distribution in wind speed mixture (IWeD). The FISTA algorithm was implemented in MATLAB and all computations were performed on a windows 64-bit personal computer with an Intel i7-3770k @ 3.5 GHz and 24 GB of memory. A detailed description, including the MATLAB code, can be obtained upon request.

Several simulated data sets were chosen to represent analytical situations. First, the set A consists of three speed wind Weibull distribution with scaled factor equal to 20 and three different kappa values 1.5 (κ_1), 3 (κ_2) and 12 (κ_3). Second, the set B consists of three speed wind Weibull distribution with shape factor equal to 10 and three different lambda values 5 (λ_1), 10 (λ_2) and 15 (λ_3). Finally, the last data set C, consists of three speed wind Weibull distribution too. In this case, we have mixed both data set A and B. We have varying the values of kappa factor and lambda factor. The values of kappa factor are 1.5 (κ_1), 3 (κ_2) and 12 (κ_3) and the values of lambda factor are 5 (λ_1), 10 (λ_2) and 15 (λ_3).

Figure 2 presents the sum of speed wind Weibull distribution of the set A for the first simulation, while Figure 3 shows the results obtained for the simulated experiment A consisting in the sum of three Speed wind Weibull distribution with values 1.5, 3 and 12 for κ_1 , κ_2 and κ_3 , when the inversion of the Weibull distribution (IWeD) is applied.

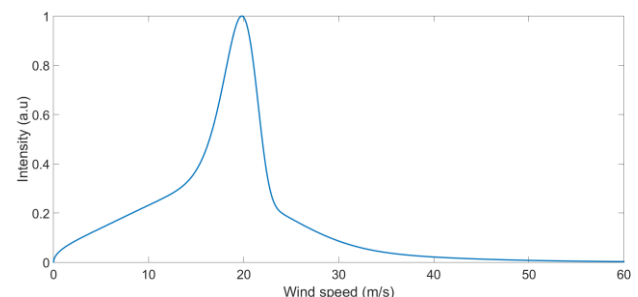


Fig. 2. Wind speed distribution composed by the sum of three Weibull distributions with scale factor equal to 20 and three different kappa values. These kappa values are 1.5, 3 and 12.

For the second simulation, Figure 4 presents the sum of speed wind Weibull distribution for the set B. The result obtained is shown in the figure 5. This shows the results obtained on the simulated experiment B consisting in the

sum of three wind speed Weibull distribution with values 5, 10 and 15 for λ_1, λ_2 and λ_3 , when it is applied the inversion of the Weibull distribution (IWeD).

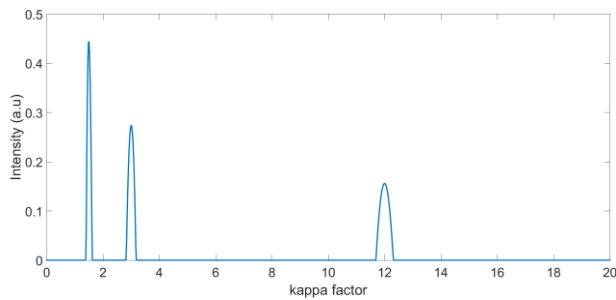


Fig. 3. Solution of the optimization release with FISTA algorithm. The kappa values detected are 1.5, 3 and 12.

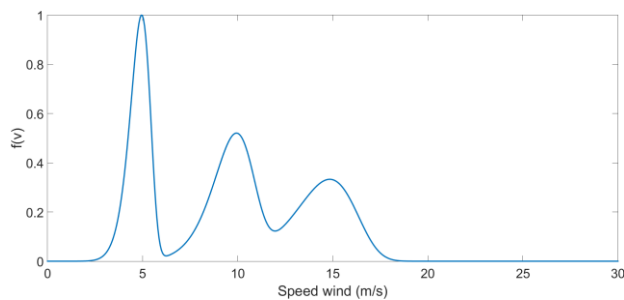


Fig. 4. Speed wind distribution composes by the sum of three Weibull distribution with shape factor equal to 10 and three different lambda values. These lambda values are 5, 10 and 15.

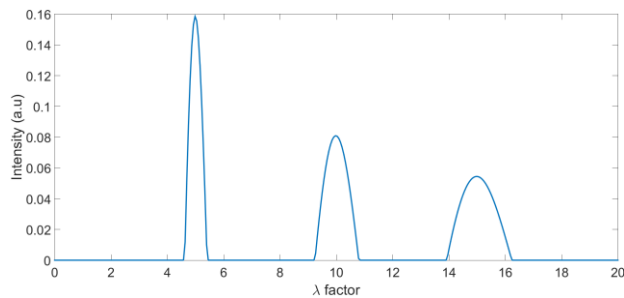


Fig. 5. Solution of the optimization release with FISTA algorithm. The lambda values detected are 5, 10 and 15.

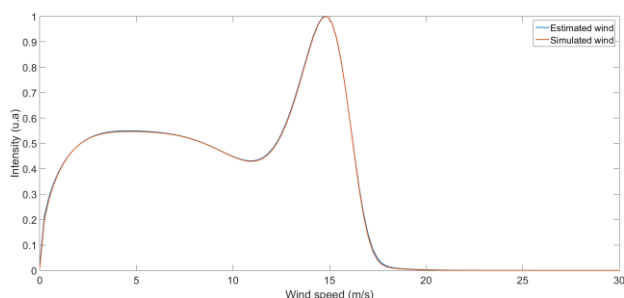


Fig. 6. Wind speed distribution composes by the sum of three Weibull distribution with three shape factors equal to 5, 10 and 15 and three different kappa values. These kappa values are 5, 10 and 15.

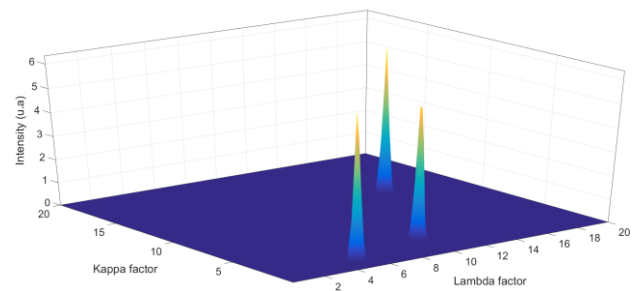


Fig. 7. Solution of the optimization release with FISTA algorithm. The lambda values detected are 5, 10 and 15. The kappa values detected are 1.5, 3 and 12.

Figure 6 presents the sum of speed wind Weibull distribution for the last simulation, set C. Figure 7 shows the results obtained on the simulated experiment C consisting in the sum of three wind speed Weibull distribution with values 1.5 (κ_1), 3 (κ_2) and 12 (κ_3) for kappa factor and 5, 10 and 15 for λ_1, λ_2 and λ_3 , for lambda factor, when it is applied the inversion of the Weibull distribution (IWeD). Figure 7 presents the values of the matrix $A(\kappa, \lambda)$. The solution can be interpreted as follows: the solution has three different mixture of winds speed, where it is possible to get the coordinates for values of kappa and lambda factors. The combination of these wind speeds obtains a solution with coordinates equal to $(\kappa, \lambda) = \{(1.5, 5), (3, 10), (12, 15)\}$.

3. Conclusion

Two and three-parameter Weibull distribution is still in development and it is probably on wind mixture where it can enhance understanding. In fact, this problem is solved for two winds mixed at most. This is crucial to understand the behavior of the wind and his theoretical power estimation. For these reasons, this paper presents the IWeD method for the analysis of winds speed mixture. The proposed method does need sparsity promote, which must be carefully chosen. This method is adequate for deconvolved winds speed mixtures. Three data sets A, B and C have been simulated to show the potential of this approach, which is able to decompose the data sets mixture of wind speed distribution in its fundamental Weibull distributions. It is concluded that IWeD method is very suitable and efficient in order to estimate Weibull parameters for wind energy applications.

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