Fast and Efficient MPC Approaches for Multilevel Drives Considering Cost Function Terms Dependency
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Abstract—Model predictive control (MPC) for multilevel converter has a multi-term cost function that requires high computation burden. In this paper, the deviation terms considered are classified into two categories according to their dependency on the voltage vector or the switching states of the multilevel converter. Subsequently, two approaches to reduce the computation burden are proposed. Mathematical models for the proposed MPC approaches are discussed and analyzed. An experimental platform for a five-level T-type multilevel induction drive is used to validate the proposed study. The experimental results show that the proposed approaches have substantially reduced the MPC computation burden compared to the conventional MPC techniques.

Keywords—Multilevel converters, Model predictive control, T-type inverter

I. INTRODUCTION

Multilevel converters (MLCs) are well-known alternative solution for the two-level voltage source converters thanks to supporting operation at high power and medium voltage levels [1]-[3]. The former has high harmonic contents, can support only low voltage applications (up to 1700 V only, as this voltage is the rating of the available market semiconductor switches). In addition, MLCs can be controlled using fundamental frequency techniques that dramatically reduces the converter losses and as a result increase the power electronic converter efficiency[4]-[6]. MLCs are classified in two main categories, according to the DC supply sharing in the three-phase circuit, which are isolated and common DC sources. Two well-known topologies are using common DC sources, which are diode clamped converter (DCC) and flying capacitor converter (FCC). The isolated DC link topology is known as cascaded full bridge converter (CFB). The latter one has the drawback of using more DC sources compared to DCC and FCC. However, it has the advantage of modularity and simplicity in implementation and control. The FCC has the drawback of requiring capacitor pre-charging as well as DC link capacitor balancing. However, the DCC requires only capacitor balancing control. Recently, many enhancements have been implemented to the different MLC topologies, among them is the T-type MLC [6]. This topology is more efficient compared to the conventional DCC as it works at lower conduction losses and has lower number of switching devices[7], [8]. Therefore, T-type MLC topologies will be considered in the proposed study.

Among of the advanced control technique is the model predictive control (MPC). The finite-control set MPC is preferred where it matches the discreet nature of the power electronic converters. This technique is based on predicting a horizon of steps for the upcoming controlled parameters using a predefined model of the electrical drives used [9]-[12]. A cost function includes the deviations of the controlled parameter is evaluated for the possible number of voltage vectors and corresponding switching states. Subsequently, the approach identifies the optimum voltage vector that minimizes the cost function and applies the corresponding switching states power electronic switches in order to achieve the control target [13]. The complexity of the MPC depends on the model under control, the number of the cost function terms and the number of the cost function evaluations [14]. Therefore, MLCs have a problem with MPC as it includes many cost function terms and has high number of switching states, in particular for high number of voltage levels, i.e. five-level, seven-level,…etc. In [14], an approach is used to reduce the number of switching states to reduce the computation burden of a five-level T-type MLC induction drive system. However, the implementation time was still high, in particular, if the number of cost function terms increased. In [13], an advanced technique is used to cascade the cost function evaluations and the approach has succeeded to reduce substantially the computation burden. However, it considered only three terms in the cost function. Many attempts have been reported to reduce the computational burden for different converter topologies, including quasi Z-source inverters [15], four-level flying capacitor converters [16], and 3-L modular MLC [17]. However, the simplification method presented in [16] reduces the degree of freedom for one of the control terms that may negatively affect the control response with the other redundant switching states. On the other hand, the number of control options becomes much complex for higher number of voltage levels or converter submodules, which limits its applicability[17].

This paper aims to identify the cost function deviations effect on computation burden of MLC MPC approaches. The identification is based on defining the cost function terms
dependency on MLCs switching state or voltage vector. Firstly, a summary for the most well-known MLC cost function terms is provided. Then, the cost function terms will be classified into two categories in order to identify their effects on the computation burden reduction. The proposed approach will be applied to the dual three-level T-type MLC driving an open-ends induction drive (OEIM). The experimental results will be discussed and analyzed. The paper remainder is organized as follows. In Section II, a mathematical model for the MPC applied to IM will be discussed. In addition, mathematical model of the MPC applied to T-type MLC will be highlighted. In Section III, the cost-function terms of the T-type MLC will be discussed. The classification of the cost function terms will be described in Section IV. The experimental results for the proposed drive system will be discussed in Section V. Finally, the paper conclusion will be derived in Section VI.

II. MODEL PREDICTIVE CONTROL OF OEIM FED BY DUAL 3L T-TYPE MLC

Considering a three-phase induction motor is connected to a five-level (dual 3L T-type MLC) described in Fig. 1. Each 3L T-type converter consists twelve semiconductor switches with antiparallel diodes.

The dynamic model of IM can be expressed using different mathematical model for the MPC applied to IM will be described. Several discretization methods are available in literature [18]. For the sake of simplicity, Euler discretization method is used. The discrete state space model can be expressed using

\[
A = \begin{bmatrix}
\frac{-1}{\tau_s} & \frac{k_r}{h_e r_e} & \frac{k_r}{h_e r_e} & \frac{-1}{\tau_s} & \frac{k_r}{h_e r_e} \\
0 & \frac{L_s}{R_s} & 0 & \frac{L_s}{R_s} & 0 \\
\frac{L_m}{\tau_r} & 0 & -\frac{1}{\tau_r} & \omega_r & -\frac{1}{\tau_r} \\
0 & \frac{L_m}{\tau_r} & 0 & -\frac{1}{\tau_r} & \omega_r \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
\frac{1}{\tau_s} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(2)

where \( R_s \) and \( R_e \) are stator and rotor resistances, \( L_s, L_r \) and \( L_m \) are stator, rotor, and mutual inductances respectively. \( \omega_r \) is electrical rotor speed. \( k_r = L_m/L_r \) is the rotor coupling factor. \( R_e = R_s + k_r R_r \) represents the equivalent resistance. \( L_\sigma = L_s (1 - L_m/L_r) \) is the transient inductance of the machine. \( \tau_s = L_s/R_s \) is the stator time constant. \( \tau_r = L_r/R_r \) is the stator transient time constant.

The electromagnetic torque can be calculated as:

\[
T = \frac{3}{2} n_p (\psi_s \times \dot{\psi}_s)
\]

(3)

where \( n_p \) is the number of pole pairs and \( \psi_s \) is the stator flux.

The prediction step in MPC requires the knowledge of the discrete model of IM. Several discretization methods are available in literature [18]. For the sake of simplicity, Euler discretization method is used. The discrete state space model can be expressed using

\[
x^{k+1} = Ax^k + Bu^k
\]

\[
A_d = I + TA
\]

\[
B_d = T_d B
\]

(4)

where \( I \) is the identity matrix and \( T_d \) is the sampling time.

B. Dual 3L T-type Inverter Output Voltage Model

The applied stator voltage can be calculated as

\[
u = u_{sat} = V_{dc}/2 (C\ell S_{cont1} - C\ell S_{cont2})
\]

(5)

where \( V_{dc} \) is the summation of the capacitor voltages. \( S_{cont1} \) and \( S_{cont2} \) are the mathematical representation of the three-phase voltage source inverter based on the switching functions \( m_x \) and \( m_{x'} \), which can be defined as [19]:

\[
S_{cont1} = \begin{bmatrix}
\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\
\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{2}{3}
\end{bmatrix}
m_x
\]

\[
S_{cont2} = \begin{bmatrix}
\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\
\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{2}{3}
\end{bmatrix}
m_{x'}
\]

(6)

\( T_{ci} \) represents Clarke transformation, which can be described as [22]:

\[
T_{ci} = \frac{2}{3} \begin{bmatrix}
0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

(7)

C. Torque and Flux Prediction

The rotor flux can be estimated from the rotor dynamics of IM expressed at the rotor reference frame as follows:

\[
\dot{\psi}_r + \tau_r \frac{d\psi_r}{dt} = L_m i_s
\]

(8)

After using Euler discretization, it can be expressed as:

\[
\psi_r = L_r i_r + L_m T_{ci} R_r \frac{d\psi_r}{dt}
\]

(9)

Knowing the rotor flux and using current measurements, (4) can be used to predict rotor flux one-step ahead. Then stator flux can be calculated at the \( k + 1 \) sample from:

\[
\psi_{s(k+1)} = k_r \psi_{r(k+1)} + L_m i_s
\]

(10)

It should be noted that the variables in (4) and (10) are expressed in stator reference frame. Therefore, appropriate
coordinate transformation should be considered. In order to compensate the time delay caused by the calculation process, the variables at sample $k+1$ can be predicted using the variables at instant $k + 1$ as follows:

$$
\begin{align*}
\dot{x}^{k+2} &= A_x \dot{x}^{k+1} + B_d u^{k+1} \\
\psi^{k+2} &= k_\psi \frac{\psi_{\text{ref}}^{k+2}}{T_{\text{rated}}} + I_{\text{sc}} L_{\text{sc}}^{k+2} \\
T^{k+2} &= k_T \frac{T_{\text{ref}}^{k+2}}{\psi_{\text{rated}}} (1 + \frac{2}{\beta} n_p) (\frac{\psi_{\text{ref}}^{k+2}}{T_{\text{rated}}} + i^2 (\frac{\psi_{\text{ref}}^{k+2}}{T_{\text{rated}}} + i^2 (1 + \frac{2}{\beta} n_p))
\end{align*}
$$

\[ (11) \]

D. Capacitor Voltage Prediction

For a capacitor, $x$:

$$
\frac{dv_{C_x}}{dt} = i_{C_x}/C_x
$$

(12)

By using Euler discretization, the predicted capacitor voltage can be expressed as

$$
v_{C_x}^{k+1} = v_{C_x}^k + \frac{T_s}{C_x} i_{C_x}^{k+1}
$$

(13)

The capacitor current can be obtained as a function of the three-phase currents and the switching states. Firstly, the capacitor currents can be expressed as a function of the currents $i_1, i_2,$ and $i_3$ (See Fig. 1) as follows:

$$
\begin{align*}
i_{C1} &= i_{det} - i_1 \\
i_{C2} &= i_{det} - i_2 \\
i_{C3} &= i_{det} - i_3
\end{align*}
$$

(14)

The objective of the controller is to maintain equal energy among the capacitors. Hence, the DC link currents $i_{det}$ as $i_{C2} = 0$ [23][24]. The capacitor currents can be written as:

$$
\begin{align*}
i_1 &= i_1 - i_3 - i_2 \\
i_2 &= -i_1 - i_3 - i_2 \\
i_3 &= i_1 - i_2 - i_3
\end{align*}
$$

(15)

The currents $i_1, i_2, i_3$ and $i_4$ can be expressed as a function of the three-phase currents by means of switching functions as follows:

$$
\begin{align*}
i_1 &= m_a i_a + m_b i_b + m_c i_c \\
i_2 &= -m_a i_a - m_b i_b - m_c i_c
\end{align*}
$$

(16)

where $m_a, m_b,$ and $m_c$ are the switching functions for phases $A, B$ and $C$, respectively; i.e. these switching functions can be set to 0 or 1. Similarly, the second horizon prediction can be performed like the flux and torque. The predicted currents can be obtained by the first relation in (11).

III. DEPENDENCY CLASSIFICATION OF COST FUNCTION TERMS

In order to make torque control of IM using MLCs, some important terms have to be considered in the cost function. These are torque deviation, flux deviation, and capacitor voltage balancing of the MLC’s DC link. Therefore, a three-term cost function is optimized in this study. In addition, the cost function terms have different weights, which requires at least two normalized weighting factors to be tuned. In addition, if some other functions need to be added, like common-mode voltage reduction, switching losses reduction or total harmonic distortion reduction, the number of cost function terms will be increased and therefore the cost function evaluation requires much computations. On the other hand, the tuning of the weighting factors becomes a challenge. Therefore, a solution for reducing the computation burden is needed. In order to reduce the computation burden, the characteristics of each cost function term are investigated.

Generally speaking, the MLCs has the advantages of high number of voltage vectors. In addition, each voltage vector has redundant switching states that can be used to optimize the cost function computation. In this study, the cost function terms is classified into two main categories according to their dependency on switching states or the corresponding voltage vectors. For instance, the machine flux and torque are the motor quantities that depend on the voltage vector, whatever the applied switching states. On the other hand, the capacitor balancing, the common-mode voltage, and the switching losses depend on the switching states. Therefore, instead of evaluating the cost functions including the overall number of cost function terms, the cost function can be divided into two cascaded functions. The first one includes the voltage-vector dependent terms and the second cost function includes the switching states dependent terms. For instance, for the five-level converters, the number of voltage vectors are 61, as described in Fig. 2. Therefore, the flux and torque deviation terms can be combined in one cost function and be evaluated only for 61 times. This cost function can be described by

$$
J_1 = \frac{|T_{\text{ref}} - T^{k+2}|}{T_{\text{rated}}} + K_T \left( \left\| \psi_{\text{ref}}^{k+2} - \psi_{\text{rated}}^{k+2} \right\| \right)
$$

(17)

where $T_{\text{ref}}$ and $\psi_{\text{ref}}^{k+2}$ are the reference torque and stator flux respectively, $T_{\text{rated}}$ is the rated torque and $\psi_{\text{rated}}^k$ is the rated stator flux magnitude. $K_T$ is the flux weighting factor. The optimum vector is calculated by:

$$
V_{\text{opt}} = \arg \min_{[v_1, v_2]} J_1(V_{k+1}^{k+1})
$$

(18)

Unlike the conventional DCC, dual 3L T-type topology has 729 switching states that can be easily selected to fulfill the capacitor balancing and the common-mode voltage reduction.

The switching states classification for the dual 3L T-type MLC according to their effect on capacitor balancing are listed in Table I. The effect is classified to three categories i.e. no, minor and major effect according to the voltage difference between the two series capacitors. Similarly, the switching state effect on the common mode voltage has been investigated in this study. The cost function of the switching state dependent terms can be described by:

$$
J_2 = \left| v_{C1}^{k+1} - v_{C2}^{k+1} \right| + \left| v_{C3}^{k+1} - v_{C4}^{k+1} \right| + K_c \left| \text{CMV} \right|
$$

(19)

where $v_{C1}^{k+1}$ and $v_{C2}^{k+1}$ are the predicted capacitor voltages; $x$ is 1-4, and $K_c$ is the common-mode weighting factor and $\text{CMV}$ is the common-mode voltage. The $\text{CMV}$ is defined as

$$
\begin{align*}
v_{\text{CM1}} &= \frac{1}{2} (v_{AO} + v_{BO} + v_{CO}) \\
v_{\text{CM2}} &= \frac{1}{2} (v_{AOD} + v_{BOD} + v_{COD})
\end{align*}
$$

(20)

$\text{CMV} = v_{\text{CM1}} - v_{\text{CM2}}$

It could be observed that four redundant switching states were sufficient to fulfill the capacitor balancing and reduce the CMV value. Therefore, the optimum switching state voltage vector is identified by

$$
V_{\text{opt}} = \arg \min_{[v_1, v_2]} J_2(V_{k+1}^{k+1})
$$

(21)

IV. MPC COMPUTATION ENHANCEMENT

Cascaded cost-function technique is proposed firstly in [13], while considering torque prediction and capacitor balancing only in the cost function. In this work, the cascaded technique is applied to torque prediction, capacitor balancing and common-mode voltage reduction. In addition, the work will be compared to the use of one cost function with reduced switching states technique to investigate the effect of the two approaches on the computation burden.

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The following describes the different methods and their effectiveness in reducing the computation burden.

### Table I

<table>
<thead>
<tr>
<th>No effect</th>
<th>High effect</th>
<th>Medium effect</th>
<th>No. of states</th>
</tr>
</thead>
<tbody>
<tr>
<td>216</td>
<td>24</td>
<td>84</td>
<td>24</td>
</tr>
</tbody>
</table>

The following describes the different methods and their effectiveness in reducing the computation burden.

### A. Approach-A: Reduced set of switching states

By testing the switching states effect on capacitor balancing and common mode voltage as given in Table I, it could be observed that

- 42 vectors, out of 61, can produce common-mode voltage with different values depending on the selected switching state.
- The zero vector “O” has some switching states that can produce zero common-mode voltage.
- 18 vectors, out of 61, produce only zero common-mode voltage.

Therefore, by testing the cost function in the conventional techniques where the cost function comprises all terms with weighting factors, the number of switching states could be reduced from 729 to 219 for common-mode voltage reduction and to 141 for common-mode voltage elimination. This could reduce the computation time using dSPACE 1103 digital controller from 5.5 ms to 140 µs and 70 µs for common-mode voltage reduction and elimination, respectively.

### B. Approach-B: Preselected switching states

A flow chart for the proposed MPC Approach-B is described in Fig. 4. The first loop is considering the voltage vector-based cost function and evaluated for 61 times, while the second loop considered the switching state based cost function. It is worth mentioning that the cascaded operation could achieve the capacitor balancing using only four redundant switching states for each voltage vector. Therefore, only 65 cost-function evaluations are required to obtain the optimum solution achieving torque, flux, capacitor balancing and common-mode voltage. This proposed approach consumes 21 µs for implementation. In addition, the common-mode voltage term could be eliminated from the cost function due to the use of the preselection common-mode voltage reduction switching states. This removes the tuning difficulty of its weighting factor.

### V. EXPERIMENTAL RESULTS

A prototype for the dual 3L T-type MLC feeding an OEIM is implemented in laboratory using SiC MOSFET discrete switches for MLC. A schematic diagram for the simulation model is shown in Fig. 5.

Experimental tests for the different case studies that have been introduced in Section IV have been conducted in laboratory. A summary for the experimental results will be discussed in the following subsection.
A. Experimental results for Approach-A

Applying the proposed Approach-A, the machine torque and flux, the capacitor voltages and the common-mode voltage are shown in Fig. 6. In this test, the weighting factor of the capacitor balancing and the common-mode voltage have been tested at zero value to reflect the effect of considering or neglecting these terms from the cost function. The last interval from t=12 s, both common-mode voltage reduction and capacitor balancing are considered. On the other hand, the common-mode voltage elimination results are shown in Fig. 7.

![Fig. 5: Schematic diagram of predictive torque control](image)

![Fig. 6: Experimental results for common-mode voltage reduction case](image)

The results declare that zero common-mode voltage could be attained along with capacitor balancing and machine torque and flux control.

B. Experimental test for preselected switching states with cascaded cost functions

Applying the proposed Approach-B, the capacitor balancing and the common-mode voltage reduction can be fulfilled as shown in Fig. 8. The technique shows that the capacitor balancing and the torque control could be achieved properly. In addition, the common-mode voltage is limited to a certain value according to the preselected switching states.

C. Evaluation of the different approaches

The current and voltage waveforms for both approaches have been analyzed using Fourier series. The total harmonic distortion for both voltage and current are listed in Error! Reference source not found. In addition, the torque and flux ripples for each approach are listed in the same table.

<table>
<thead>
<tr>
<th>Approach</th>
<th>THD, [%]</th>
<th>THD, [%]</th>
<th>T_rip, [%]</th>
<th>ψ_rip, [%]</th>
<th>time, [µs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) common-mode voltage elimination</td>
<td>6.45</td>
<td>27.75</td>
<td>68.48</td>
<td>3.33</td>
<td>70</td>
</tr>
<tr>
<td>(A) common-mode voltage reduction</td>
<td>2.65</td>
<td>19.60</td>
<td>34.44</td>
<td>2.41</td>
<td>140</td>
</tr>
<tr>
<td>(B) common-mode voltage reduction</td>
<td>2.33</td>
<td>17.85</td>
<td>25.65</td>
<td>1.35</td>
<td>21</td>
</tr>
</tbody>
</table>

It could be observed that the torque and flux ripples for approach-A are lower in case of common-mode voltage reduction compared to those of common-mode voltage elimination. On the other hand, it can be clearly shown that the cascaded approach could substantially reduce the computation time. In addition, by comparing evaluation values for Approach-A and B, the values of Approach-B are lower than that of Approach-A due to the reduced execution time.

![Fig. 7: Experimental results for common-mode voltage elimination case](image)

I. Conclusion

In this paper, cost function deviation terms have been identified and classified based on dependency on voltage vector and switching state. Two different approaches have been proposed and implemented. These are selecting reduced switching states strategy and cascaded cost function approaches. The two proposed approaches could substantially reduce the MPC computation. The approaches have considered torque prediction along with capacitor balancing and common-mode voltage reduction/elimination. The results show that the use of cascaded cost function approach could substantially reduce the torque and flux ripples compared to the reduced switching approach.
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