Conditions for the optimality of the criterion programming problem and determination of basic parameters of community power generation

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Abstract. The paper presents the application of the criterion programming problem and determination of its basic parameters to predict the behavior of an energy community in Ukraine during the war and post war restructure process.

The main problem of the criterion programming is the implementation of renewable energy sources – especially grid-on photovoltaic hybrid system. The theoretical method starts from the basic equation of the criterion programming problem. The model is than developed with continuous indexing resulting to orthonormal equations. Next, the convexity of the function is analyzed and system of nonlinear equations is generated.

The basic criteria is calculated what results in definition of remaining similarity criteria. Next, normalization and residual vectors are calculated allowing the definition of the optimal values. If the similarity criteria is optimal, the substitution in the equation leads to final result.

Proposed method was verified on model situation of specific community in Ukraine. The calculations were made in PV*SOL Premium as described in the last part of the paper.

This mathematical model can be practically used to analyze the energy needs of a community in emergency situation and to define the optimal energy covering of it needs. Particularly, it can be used in war and post war conditions in Ukraine to enhance the conditions in damaged and overloaded energy grid and to improve the energy resilience.

Key words. Energy community, renewable energy sources, photovoltaic system, criterion optimization, objective function.

1. Introduction

When the Russian aggression attack started against Ukraine, unpredictable challenges to the country’s security and economy, but also to its environmental protection, power engineering, green energy and sustainable and low-carbon development [1, 2] started. The energy security of Ukrainians is severely threatened for more than 2 years. Approximately 50\% of Ukraine’s energy infrastructure has been destroyed or temporarily occupied. One of the possible solutions how to improve the situation is implementation of communal power engineering [3, 4].

But in Ukraine, there is still no clear simple and efficient system for the establishment, organization and operation of an energy community. Government and non-government organizations (such as Synergy, Ecoclub, etc.) are already dealing with this problem and the first projects will be realized soon [4, 5, 6]. But many problems must be solved for efficient renewable energy source implementation in community energy in these specific conditions [7, 8, 9].

Mathematical models describing the structure of energy community and RES (photovoltaic system) operation requires to define the conditions for optimization [10, 11]. These conditions must respect specific features, exactly technical, metrological, economic, legislative and social [12, 13, 14]. The paper describes features of mathematical model using common criterion optimality [15].

2. Conditions for the optimality of the criterion programming problem and determination of basic parameters

Criterion programming (CP) as an optimization method and as an integral part of similarity theory is designed to study optimization processes in complex dynamical systems. Within the limits of real technical possibilities, the method of optimal control finds limit conditions of these systems. The method further develops the fastest direction and obtains general optimality conditions in the form of stable patterns. The relationships between optimal values of objects and phenomena is than created for appropriate automatic control systems. Therefore it is important to study the conditions for determining the optimal similarity criteria. In the CP task are not only certain dimensionless combinations of parameters from the theory of similarity, but also variable patterns of the dual optimization task [16, 17].
The relationship between the similarity criteria can be investigated from the conditions of optimality, but not directly from the numerical values of the similarity criteria. In this sense, these relations are quasi-conditions and they can serve as the basis for the development of the algorithm calculating the optimal values of similarity criteria.

If the conditions for the optimality of the CP problem are obtained, the direct problem of CP can be formulated as following equations (1) and (2).

The minimal value of similarity criteria is defined:

\[ y(x) = \sum_{i} a_{i} \prod_{j} x_{j}^{a_{ij}} \]  

under conditions

\[ g_{k}(x) = \sum_{i} a_{i} \prod_{j} x_{j}^{a_{ij}} \leq G_{k}, \quad k = 1, p. \]

where \( y(x) \) is the criterion of optimality; \( a_{i} \) are constant coefficients; \( x_{j} \) are variable parameters; \( G_{k} \) are numerical values of the limits \( g_{k}(x) \), which determine the allowable area of existence of the optimality criterion.

The continuous indexing of components of objective functions in expressions (1) - (2) is accepted for more simple definition of restrictions. The definition is \( m_{1} \) for the number of members in the objective function and only generalized in the form (3).

\[ m = m_{1} + \sum_{k} m_{k} \]  

Equation (4) defines mathematical model of components, where \( m_{k} \) is the number of members in the \( k \) constraint. The corresponding calculations show the dual problem of criterion programming.

The maximum can be calculated (4):

\[ d(\pi) = \prod_{i} a_{i} \prod_{k} \frac{\lambda_{k}}{G_{k}} \]  

from the condition of orthogonality (5):

\[ \sum_{i} a_{i} \pi_{i} = 0, \quad s = 1, n, \]  

and the condition of ratio (6):

\[ \sum_{i} \pi_{i} = 1, \]  

where \( \pi_{i} \) is the similarity criteria and \( \lambda_{k} \) are normalized Lagrange multipliers (7).

\[ \lambda_{k} = \sum_{i=m_{k}+1}^{m_{k+1}} \pi_{i} \]

It shows that the similarity criteria in the space described by the system of orthonormal equations are determined by equation (8):

\[ \pi = \beta_{0i} + \sum_{j=1}^{s} \beta_{ji} \cdot \pi_{j} \]  

where \( \beta_{0i} \) is the normalization vector; \( \beta_{ji} \) is the residual vector; \( \pi_{j} \) is basic similarity criteria and \( s = m-n-1 \) is the complexity of the criterion programming task.

Fig. 1 shows an example of the similarity criteria dependence on the problem of CP with \( s = 1 \). The end of the vector \( \pi \) always lies on the line 1-1 and determines such values of similarity criteria that satisfy the conditions (4) - (5). One of the positions of the vector \( \pi \) (it is shown on Fig. 1) gives such values of similarity criteria that provide the maximum value of the dual function (3) and the minimum value of the function (1). Actually, the problem of CP is to find such a position of the vector \( \pi \) in space, which is determined by practical conditions.

If express the similarity criteria through the basic criteria and vectors of normalization and residuals. The dual function will be rewritten as follows (9):

\[ d(\pi) = \prod_{i=1}^{m} \beta_{0i} \prod_{i} \sum_{j} \beta_{ij} \cdot \pi_{j} \]

\[ \times \prod_{k=1}^{s} \left( \sum_{i} \beta_{0i} + \sum_{j} \beta_{ij} \cdot \pi_{j} \right) \]

where \( T_{k} \) is a set consisting of the indices defining members of the \( k \) constraint.

Fig 1.: Dependence of similarity criteria.

The tasks is considered where the function \( d(\pi) \) is convex. It is always possible to replace the definition \( d(\pi)_{\text{max}} \) with the definition of a stationary point of the function, because the sets of maximal points of these functions coincide. The logarithm transformation is shown in the expression (10).
\[
\ln d(\pi) = \ln \prod_{i=1}^{m} d_{ij} = \sum_{i=1}^{m} \ln d_{ij} = \sum_{i=1}^{m} \left( \beta_i + \sum_{j=1}^{n} \beta_{ij} \cdot \pi_j \right) \times \\
\times \ln \left( \beta_i + \sum_{j=1}^{n} \beta_{ij} \cdot \pi_j \right) = \sum_{i=1}^{m} \ln \beta_i + \sum_{i=1}^{m} \sum_{j=1}^{n} \beta_{ij} \cdot \pi_j \times \ln \left( \beta_i + \sum_{j=1}^{n} \beta_{ij} \cdot \pi_j \right)
\]

(10)

Take the derivative of the function \(\ln d(\pi)\) according to the basic similarity criteria \(\pi_i\) and equate them to zero. A system of equations is obtained from which the conditions of the maximum of the function \(d(\pi)\) can be calculated from the expression (11):

\[
\prod_{i=1}^{m} \left( \beta_{io} + \sum_{j=1}^{n} \beta_{ij} \pi_j \right)^{\beta_{io}} = \prod_{i=1}^{m} \left( \sum_{j=1}^{n} \left( \beta_{io} + \sum_{j=1}^{n} \beta_{ij} \pi_j \right)^{\beta_{io}} \right)^{\beta_{io}} = \prod_{i=1}^{m} \pi_{io}^{\beta_{io}}
\]

(11)

The system in general nonlinear equations is obtained. It determines the conditions for the optimality of the problem CP. Solving it with respect to the basic variables \(\pi_i\) allows us to determine the maximization vector \(\pi_{\text{max}}\), and hence the maximum of the dual function \(d(\pi)_{\text{max}}\).

Note that the similarity criteria, which by definition are some relations of parameters, according to conditions (7) are determined without defining the variables of the direct problem CP \(x_i\). It means that the criterion model of the process, in which the found criteria of similarity \(\pi_i\) are used, establishes a complete mathematical analogy, which is invariably preserved for all possible values of variables \(x_i\) that satisfy conditions (2).

Then it can be argued that (7) are quasi-conditions for the optimality of the CP problem, which do not directly give an optimal solution, but they can be easily used to construct practical algorithms for finding the optimal solutions. The method is shown and explained in following paragraphs.

When forming a system of optimality conditions (7), the problem of allocating basic similarity criteria \(\pi_i\) arises. The composition of the basic similarity criteria is often determined by the characteristics of the process under study and in the function of which the optimal control is carried out. If such considerations do not exist, then formally this problem is solved by choosing from a set of similarity criteria for which the corresponding normalization vectors are equal one to each other and the residual vectors are zero.

Having determined the system of basic criteria, the remaining similarity criteria can be determined according to (11) using normalization vectors \(\beta_i\) and residuals \(\beta_j\). In order to obtain the vectors \(\beta_i\) and \(\beta_j\), the properties of the linear space are used. They can be described by a matrix of exponents \(\alpha\), to equivalent transformations. Convert the matrix of \(\alpha\) according to the Gauss-Jordan algorithm to the form (12).

\[
\alpha = \left[ \begin{array}{cccc} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \end{array} \right] \alpha_{n+1} \alpha_{n+2} \ldots \alpha_{m} = \left[ \begin{array}{cccc} \alpha'_{n+1} & \alpha'_{n+2} & \ldots & \alpha'_{m} \end{array} \right]
\]

(12)

Normalization vectors can be obtained from any column vector of the matrix \(\beta\). To do this, the selected vector-column of \(\beta\) must be divided by the sum of its first \(m_1\) components. As a result, the following vector (13) is obtained:

\[
\beta_o = \frac{1}{\sum_{j=1}^{m_1} \beta_{oj}} \beta_i
\]

(13)

This satisfies both the condition of orthogonality resulting from the construction of the matrix \(\beta\) and the condition of normalization, since all components of the vector \(\beta_i\) are divisible by the sum of its first \(m_1\) components.

To obtain the residual vector \(j\), it is necessary to subtract from the \(j\) vector-column of the matrix \(\beta\) the product of the sum of the first \(m_1\) of its components on the vector of normalization \(\beta_o\). It gives the residual vectors (14):

\[
\beta_j = \beta_j - \sum_{i=1}^{m_1} \beta_{ii} \cdot \beta_o, \quad j \in \{1, 2, \ldots, s + 1\}, \quad j \neq i
\]

(14)

In order to determine the optimal values of the similarity criteria from the set of their allowable values, the equation (7) is converted by dividing both its parts by \(\prod_{i=1}^{m} d_{ij} \beta_{ij}\) to get the following (15):

\[
\forall d(\pi) = \frac{\prod_{i=1}^{m} \left( \beta_{io} + \sum_{j=1}^{n} \beta_{ij} \cdot \pi_j \right)^{\beta_{io}}}{\prod_{i=1}^{m} \pi_{io}^{\beta_{io}}} \frac{1}{\prod_{i=1}^{m} \prod_{j=1}^{n} \left( \beta_{io} + \sum_{j=1}^{n} \beta_{ij} \cdot \pi_j \right)^{\beta_{io}}} = \prod_{i=1}^{m} \prod_{j=1}^{n} \left( \beta_{io} + \sum_{j=1}^{n} \beta_{ij} \cdot \pi_j \right)^{\beta_{io} \pi_j}
\]

(15)

If the similarity criteria is optimal then after substituting them in (15), the equation is equal to the value one. The iterative process of searching the optimum values of criteria similarity has structural and logical scheme presented on Fig. 2. Giving the first approximation of the similarity criteria, this may be the arithmetic mean between the respective vectors of normalization.

The values \(d(\pi)^{(0)}\) are calculated and the composition of the basic similarity criteria is selected so that all \(s\) values \(d(\pi)^{(0)}\) are smaller than 1. This provides the necessary conditions for the convergence of the iterative process.

In the next step, the values of the basic similarity criteria \(\pi_i\) are specified, reducing their values by \(d(\pi)_j\). Than the new values are calculated for that. Since these values \(\pi_i\) are greater than 1, the values for subsequent iterations are
calculated as the arithmetic mean between the corresponding basic similarity criteria of the previous iterations.

In subsequent iterations, the value $\nabla d(\pi_j)$ serves as a guiding factor and determines the direction of movement to the optimal values of the similarity criteria.

The iterative process is terminated if the following conditions in equation (16) are met:

$$\left| \nabla d\left(\pi_j^k\right) - 1 \right| \leq \varepsilon_1$$  \hspace{1cm} (16)

The coefficients $\varepsilon_2$ must meet the condition formed in equation (17):

$$\frac{|d\left(\pi_j^k\right) - d\left(\pi_{j+1}^k\right)|}{d\left(\pi_{j+1}^k\right)} \leq \varepsilon_2$$  \hspace{1cm} (17)

where $\varepsilon_1$, $\varepsilon_2$ are given from the measure of the accuracy required for the calculations.

3. Model verification on community photovoltaic power plant energy supply

The proposed criterion model was tested on simulated power generation covering the energy needs of Ukrainian community in small town with historical center and communal sporting facilities. The real location cannot be described because of strategic reasons during the war time.

Sample simulation using the photovoltaic system installed on the roof of the communal sporting hall is presented on the Fig. 3. The design of PV system respects the fire safety requirements, lightning protection rules (safe distance) and maximizes the generated energy on available surfaces not affected by shadowing (green areas on the simulation result).

Fig 2.: Structural and logical scheme of the algorithm for determining the similarity criteria

Fig 3.: Simulation on the model of the communal sporting facility (PV*SOL Premium).

Fig. 4. presents 1 sample of typical load profiles used for simulations. It is basic load chart describing annual behavior of family living in separate low energy property (shelter) with annual consumption 3012 kWh. Group of similar profiles is used for simulation of the community consumption.

Fig 4.: Sample load profile with annual consumption 3012 kWh (PV*SOL Premium).

Fig. 5 demonstrates the simulated power generation in the model community before applying the method and reaching the optimal similarity criteria according to equations (10) and (11). The blue peaks mean the overproduction not used in the community and injected to the main grid. Optimization of this energy is the task for the criterion programming.

Fig 5.: Power generation before reaching the optimal values of the similarity criteria (PV*SOL Premium).
Table I shows average power generation and the amount of underproduction. This energy covering the load chart could be used from storage (for example in high temperature battery), can cover charging electric vehicles (with or without WTG technology [18] or could be decrease using the optimization (6), (7) and (8). For purposes of this paper we expect the optimization to reach the decrease of PV system size and thus economic benefits, respectively the lower purchasing costs for better realization of the system in the war time. From this reason, the second hand modules produced 7 – 8 ago available from closed and decommissioned projects are used for simulations [19]. These panels are very often offered for reasonable prices for Ukrainian marked from Germany and still can offer acceptable parameters [20].

Table I: Power production and load coverage before optimization of the photovoltaic system.

<table>
<thead>
<tr>
<th>Load</th>
<th>PV system</th>
<th>Grid</th>
<th>month</th>
<th>annual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kWh</td>
<td>kWh</td>
<td></td>
<td>%</td>
</tr>
<tr>
<td>Jan</td>
<td>2216,4</td>
<td>124,4</td>
<td>2092,8</td>
<td>5,612705</td>
</tr>
<tr>
<td>Feb</td>
<td>1727,4</td>
<td>208,3</td>
<td>1519,8</td>
<td>12,05859</td>
</tr>
<tr>
<td>Mar</td>
<td>1240,1</td>
<td>227,1</td>
<td>1033,7</td>
<td>18,31304</td>
</tr>
<tr>
<td>Apr</td>
<td>614,8</td>
<td>195,5</td>
<td>419,8</td>
<td>31,79896</td>
</tr>
<tr>
<td>May</td>
<td>189,1</td>
<td>83,3</td>
<td>106,4</td>
<td>44,05077</td>
</tr>
<tr>
<td>Jun</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Jul</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Apr</td>
<td>12,7</td>
<td>5,9</td>
<td>7,4</td>
<td>46,45669</td>
</tr>
<tr>
<td>Sep</td>
<td>197,9</td>
<td>79,6</td>
<td>118,6</td>
<td>40,22233</td>
</tr>
<tr>
<td>Oct</td>
<td>572,5</td>
<td>151,9</td>
<td>421,3</td>
<td>26,53275</td>
</tr>
<tr>
<td>Nov</td>
<td>1275,6</td>
<td>125</td>
<td>1151,4</td>
<td>9,79931</td>
</tr>
<tr>
<td>Dec</td>
<td>195,7</td>
<td>108</td>
<td>1846,6</td>
<td>5,227973</td>
</tr>
<tr>
<td>sum</td>
<td>10000,2</td>
<td>1309</td>
<td>8698,9</td>
<td>13,089782</td>
</tr>
</tbody>
</table>

Fig. 6 presents situation for model Variant 1 after the optimization process. The gray columns have the same structure like on Fig. 4, while the second aggregated columns mean the summary of coverage from own communal photovoltaic plant, external grid and battery storage.

Table II shows 4 variants of the same situation after the optimization process. The similarity vectors describing the power generation, power production and power consumption in equations (1) and (2) are recalculated according to the flow chart on Fig. 2.

Table II: Optimization process (4 variants).

<table>
<thead>
<tr>
<th>Variant 1</th>
<th>Variant 2</th>
<th>Variant 3</th>
<th>Variant 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>south</td>
<td>e/w</td>
<td>south</td>
<td>e/w</td>
</tr>
<tr>
<td>kWh</td>
<td>kWh</td>
<td>kWh</td>
<td>kWh</td>
</tr>
<tr>
<td>Jan</td>
<td>331,8</td>
<td>130,8</td>
<td>67,3</td>
</tr>
<tr>
<td>Feb</td>
<td>393,9</td>
<td>235,6</td>
<td>537,7</td>
</tr>
<tr>
<td>Mar</td>
<td>778,6</td>
<td>519,8</td>
<td>890,3</td>
</tr>
<tr>
<td>Apr</td>
<td>583,4</td>
<td>810,7</td>
<td>1037,6</td>
</tr>
<tr>
<td>May</td>
<td>1130,1</td>
<td>1046,5</td>
<td>1339,9</td>
</tr>
<tr>
<td>Jun</td>
<td>1133,8</td>
<td>1087,3</td>
<td>1141,4</td>
</tr>
<tr>
<td>Jul</td>
<td>1179,4</td>
<td>1081,8</td>
<td>1161,4</td>
</tr>
<tr>
<td>Aug</td>
<td>1008,3</td>
<td>940,4</td>
<td>1133,4</td>
</tr>
<tr>
<td>Sep</td>
<td>825,4</td>
<td>610,8</td>
<td>852,8</td>
</tr>
<tr>
<td>Oct</td>
<td>573</td>
<td>327,9</td>
<td>587,8</td>
</tr>
<tr>
<td>Nov</td>
<td>293,1</td>
<td>103,3</td>
<td>325,2</td>
</tr>
<tr>
<td>Dec</td>
<td>209</td>
<td>52,1</td>
<td>257,5</td>
</tr>
<tr>
<td>sum</td>
<td>8966,8</td>
<td>6934,8</td>
<td>9423,5</td>
</tr>
<tr>
<td>optim</td>
<td>6860,7</td>
<td>5782,68</td>
<td>7142,04</td>
</tr>
<tr>
<td>7262,39</td>
<td>6194,61</td>
<td>7228,99</td>
<td>6018,66</td>
</tr>
</tbody>
</table>

It is evident that the difference between original annual sum of energy flow compared to optimized variants is between 1000 – 2000 kWh what means difference between 15 – 20 %. Although it could be expected that east / west system should give better results, in this case the south oriented system is more profitable. The reason is rather specific load chart for the refugees shelters included in the energy community.

4. Results and Conclusions

The simulations summarized in Table II show that interesting results can be expected after usage of criterion programming for optimization of energy flows inside the energy community. The algorithm was applied on specific energy community situated in Ukraine and containing also emergency shelters for war refugees. These objects have rather specific features and load chart because they are designed with view on secondary applications after the war for another communal purposes, sporting or cultural events.

Fig 6.: Covering the energy needs after the optimization – Variant 1 (PV*SOL Premium).

3 other variants were simulated with similar results. Every variant was calculated with photovoltaic array oriented to south with inclination 35° and with dual system east / west with inclination 15°.

Fig 6.: Covering the energy needs after the optimization – Variant 1 (PV*SOL Premium).

Table 2: Optimization process (4 variants)

References